PDE and Wavelet Techniques for Image Compression

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SCPDE02, Baptist Univ., Hong Kong, Dec 12-15, 2002

Collaborator: Hao-Min Zhou (Caltech)

Reports: www.math.ucla.edu/applied/cam/index.html
or www.acm.caltech.edu/~hmzhou

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Wavelet Transforms and PDE based Techniques in Image Processing

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Outline

¥Introduction & Motivations

¥ENO-Wavelet Transforms

¥Application in Image Compression

¥Total Variation (TV) Model for Wavelet Thresholding

¥Conclusion

Motivations

¥ Wavelets: great impact in image processing

¥ PDEs: increasingly effective in image processing

Our goal: combine best of both techniques

Introduction

Typical image processing tasks:

¥Restoration (denoising, deblurring)
¥Enhancement
¥Compression
¥Segmentation
¥Patent Recognition
¥Still v.s. Video

Applications:

Medical, Biotech, Physical Science, Astronomy, Law Enforcement, Environment, Entertainment, Military, Chemistry, ...

Introduction

¥ Wavelets representation (Harmonic analysis)

given $\psi(x) \longrightarrow \psi_{j,k}(x) = 2^{\frac{j}{2}}\psi(2^j x - k) \longleftarrow$ Orth. basis

$$f(x) = \sum c_{j,k}\psi_{j,k}(x), \quad c_{j,k} = \int f\psi_{j,k}$$

Low Freq. (average)

High Freq. (difference)

¥ Wavelets in image compression.

¥Good features:

¥Orthonormal basis

¥Concentrate energy

¥Approximate smooth function efficiently

¥High order of accuracy

¥Multiresolution

¥Fast transform algorithms

¥Limitation: Oscillations at discontinuities

Introduction

¥ PDE s in Image Processing

¥New alternative to FFT/wavelets and stat. approaches

¥Treats images as piecewise continuous functions connected by edges

¥Use PDE concepts: gradients, diffusion, curvature, level sets

¥Variational: Euler-Lagrange gives PDE, e.g. TV denoising

$$\min \int |\nabla u| \quad \text{S.T.} \quad \| u - u_0 \| \le \sigma \longrightarrow \nabla \left(\frac{\nabla u}{|\nabla u|} \right) - \lambda (u - u_0) = 0$$

¥Advantages:

¥Sharper edges,

¥better geometric properties

¥exploit sophisticated PDE and CFD techniques: Hamilton-Jacobi, shock capturing

Motivations

¥Avoiding Gibbs phenomenon:

Oscillations at discontinuities.

¥Reason for Gibbs:

Discontinuities $\longrightarrow$ Large high freq.

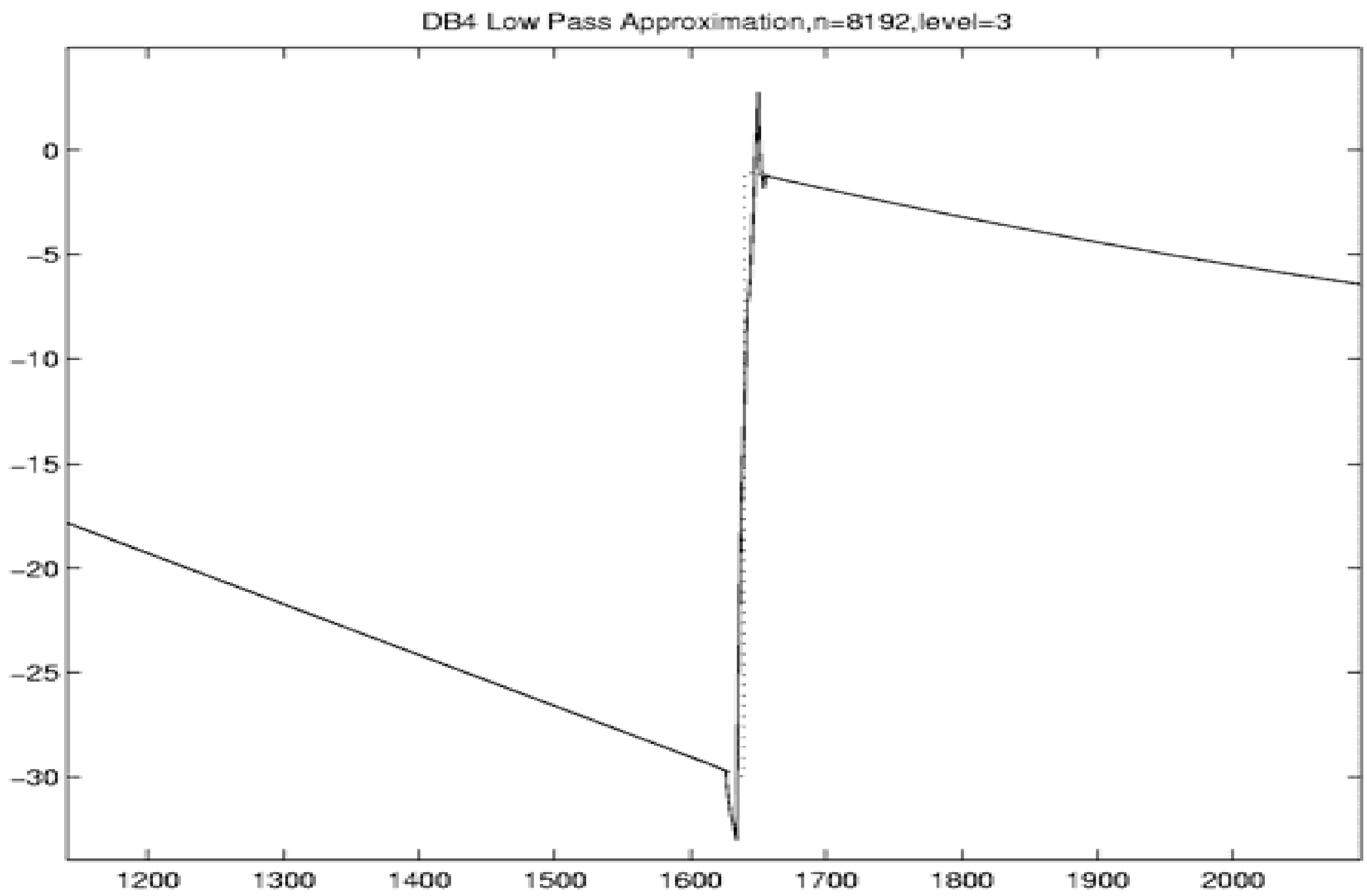
Truncate high freq. $\longrightarrow$ Destroy discontinuities

$\longrightarrow$ Generate Oscillations.

¥Examples:

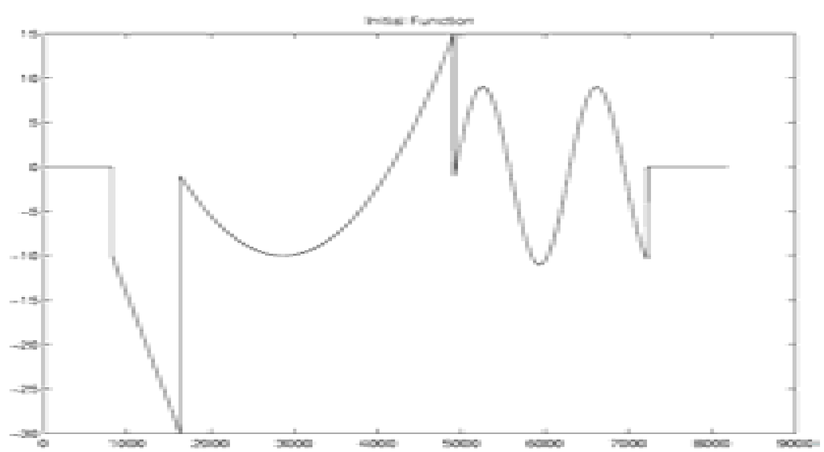
- **Fourier: well known.**
- **Wavelets: Better (more local) but still there.**

Motivations …

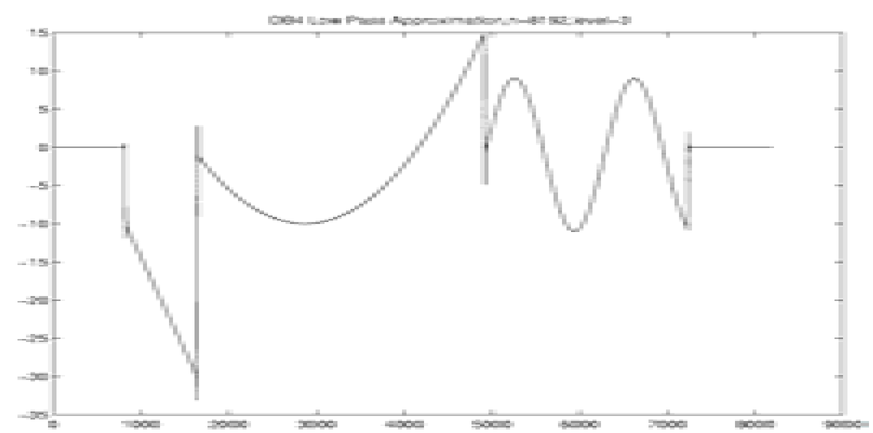


Gibbs oscillations at a discontinuity

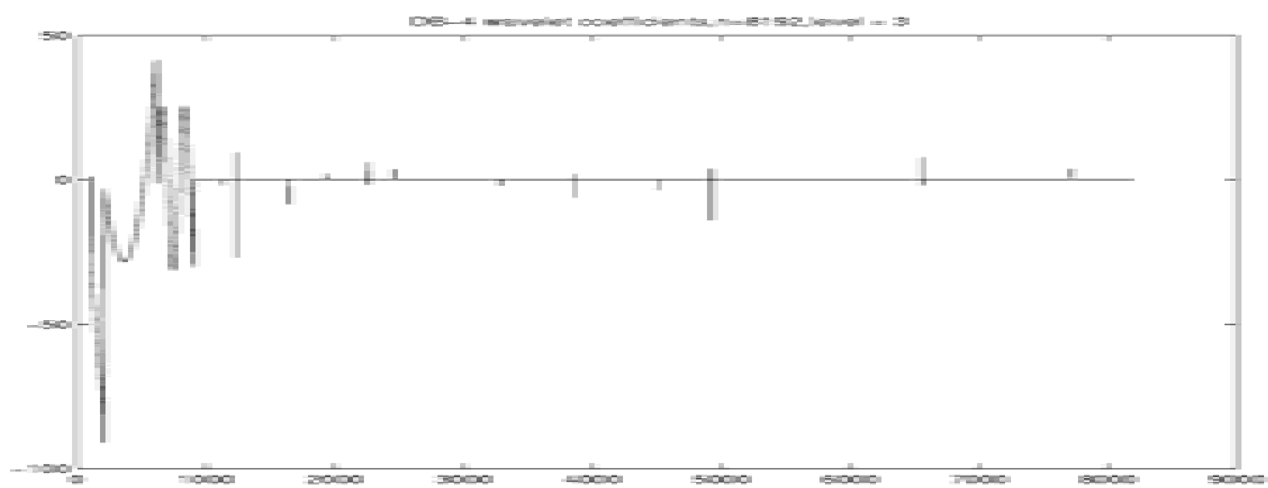
Motivations ...



Original function

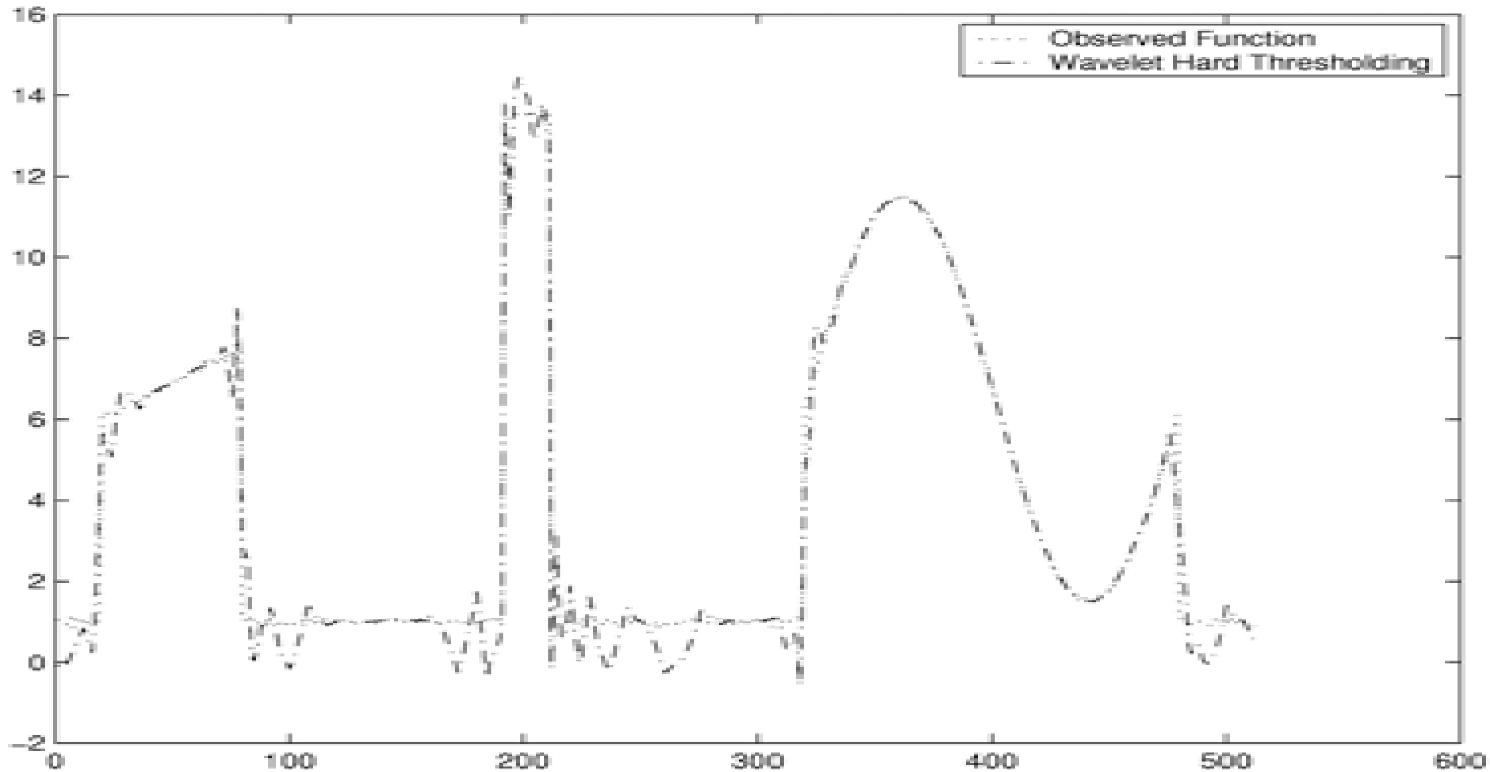


3-level DB4 approximation



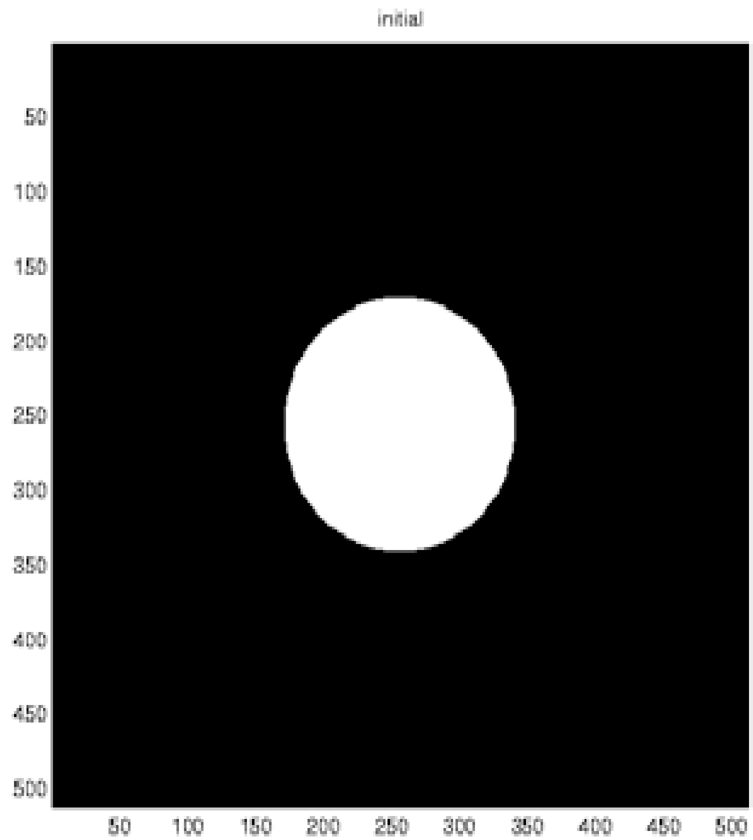
3-level DB4 coefficients, large high frequencies corresponding to jumps

Motivations ...

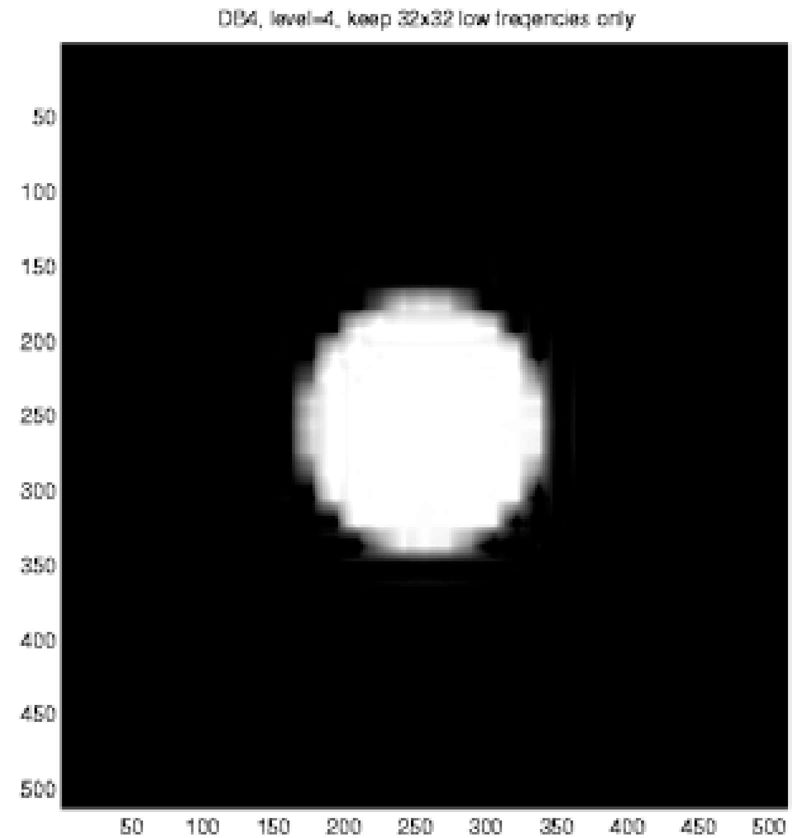


Gibbs for noisy data

2-D Gibbs



Original 2-D Function



4-level DB4, Gibbs oscillations at edges

Gibbs leads to poor results

¥ Approximation error

¥ Denoising: Edge smearing and oscillations

¥ Compression: worse with same ratio

Motivations

¥ Using PDE techniques in wavelets:

¥Modify wavelet transforms: Adaptive ENO-Wavelet Transforms (ENO schemes are very popular and successful in CFD), such that no large high freq. coefficients are generated.

¥Modify the standard wavelet coefficients: Total-Variation (TV) based wavelet image compression (TV leads to nonlinear PDE s)

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¥Total Variation (TV) Model for Wavelet Thresholding

¥Conclusion

Topic 1

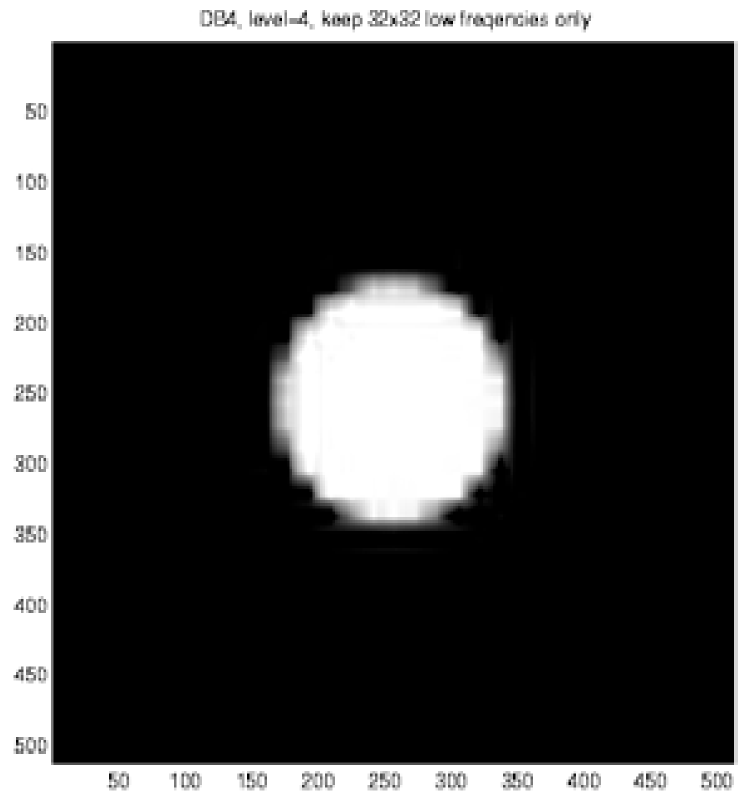
ENO-wavelet transform and its application in image compression

Goals

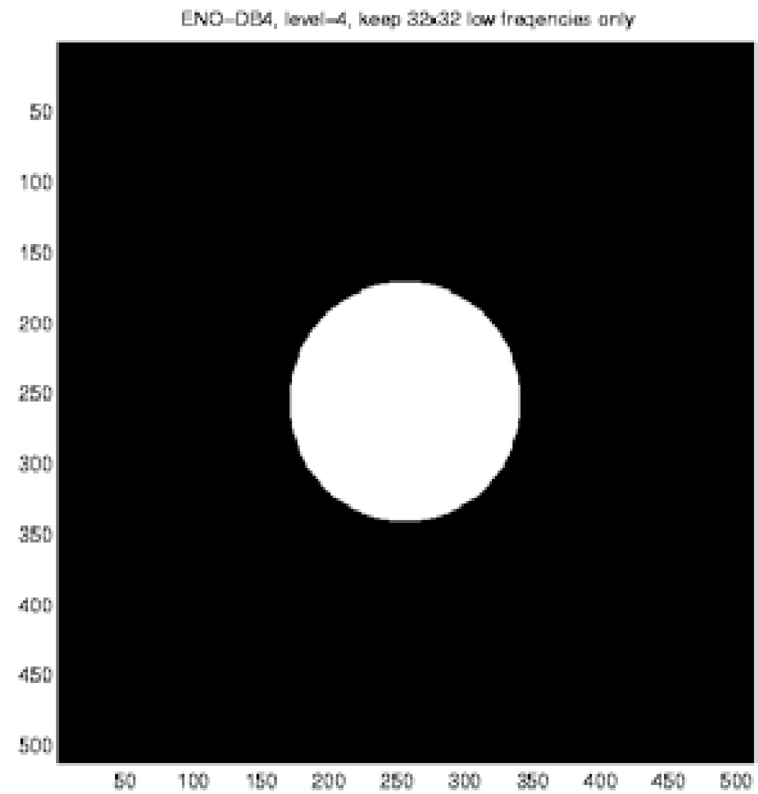
Modify standard wavelet transforms to have the following properties:

1. Essentially Non-Oscillatory (ENO).
2. Retain Pyramidal filtering framework.
   — *Functional replacement of existing wavelet transforms*
3. Error bound depends only on derivatives away from discontinuities.
4. Minimal extra cost and storage.
   — *Proportional to number of discontinuities.*

Goals ...



DB4, 4-level



ENO-DB4, 4-level

Pyramidal Wavelet Transforms

$$\begin{array}{ccccc} \alpha_{j+1,i} & \longrightarrow & \alpha_{j,i} & \longrightarrow & \alpha_{j-1,i} \\ & \searrow & & \searrow & \cdots \\ & & \beta_{j,i} & & \beta_{j-1,i} \end{array}$$

Consider p vanishing moments wavelets: $p=(l+1)/2$

¥Low freq. (average):

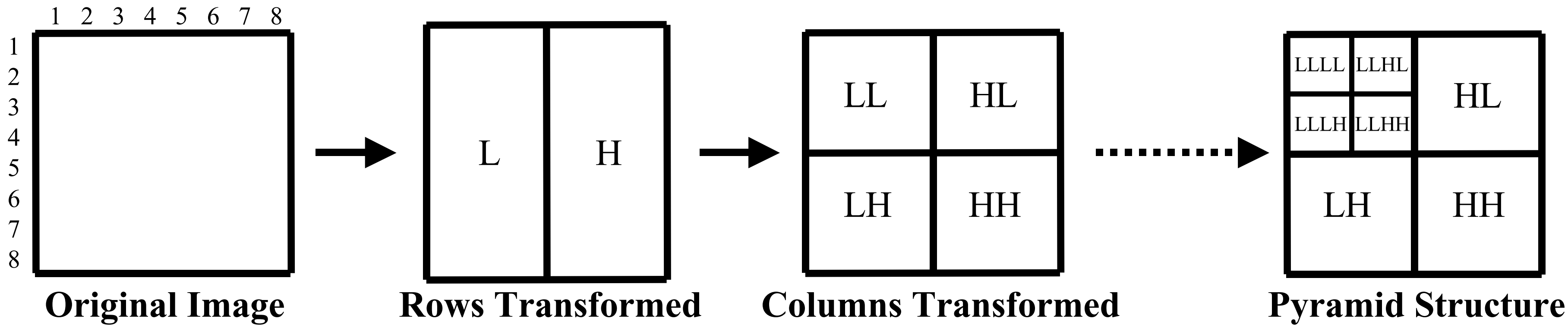
$$\alpha_{j,i} = \sum_{s=0}^{l} c_s \alpha_{j+1,2i+s}$$

¥High freq. (p-th order deriv.):

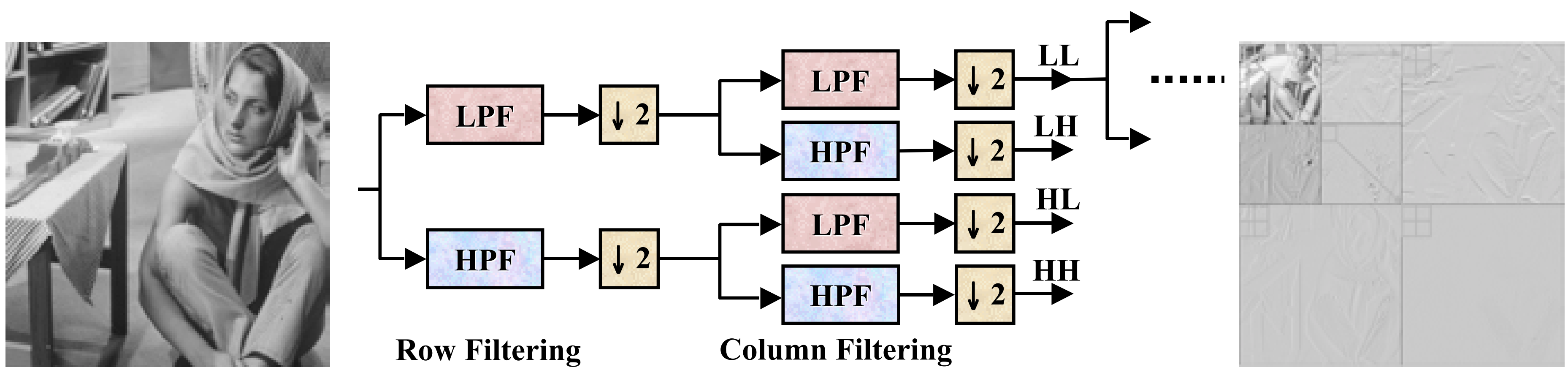
$$\beta_{j,i} = \sum_{s=0}^{l} h_s \alpha_{j+1,2i+s}$$

¥**Jump in** α_{j+1} $\longrightarrow$ **Large** β_j

Wavelets



1 2 3 4 5 6 7 8
1 2 3 4 5 6 7 8
L
H
LL
HL
LH
HH
LLLL
LLHL
LLLH
LLHH
HL
LH
HH
Original Image
Rows Transformed
Columns Transformed
Pyramid Structure



LPF
HPF
↓ 2
↓ 2
LPF
HPF
LPF
HPF
↓ 2
↓ 2
↓ 2
↓ 2
LL
LH
HL
HH
Row Filtering
Column Filtering

Approaches

¥All linear transforms:

¥Gibbs Oscillations

¥Must use data-adaptive nonlinear transforms

¥Thresholding (Hard and Soft):

¥Donoho, DeVore, Daubechies

¥Limitation: Complicated data structure to record locations of large high freq.

¥Geometry and Wavelets:

Special basis to represent discontinuities

¥Candes and Donoho: Rigdelets and Curvelets.

¥Mallat and Collaborators: Bandelets

Approaches ...

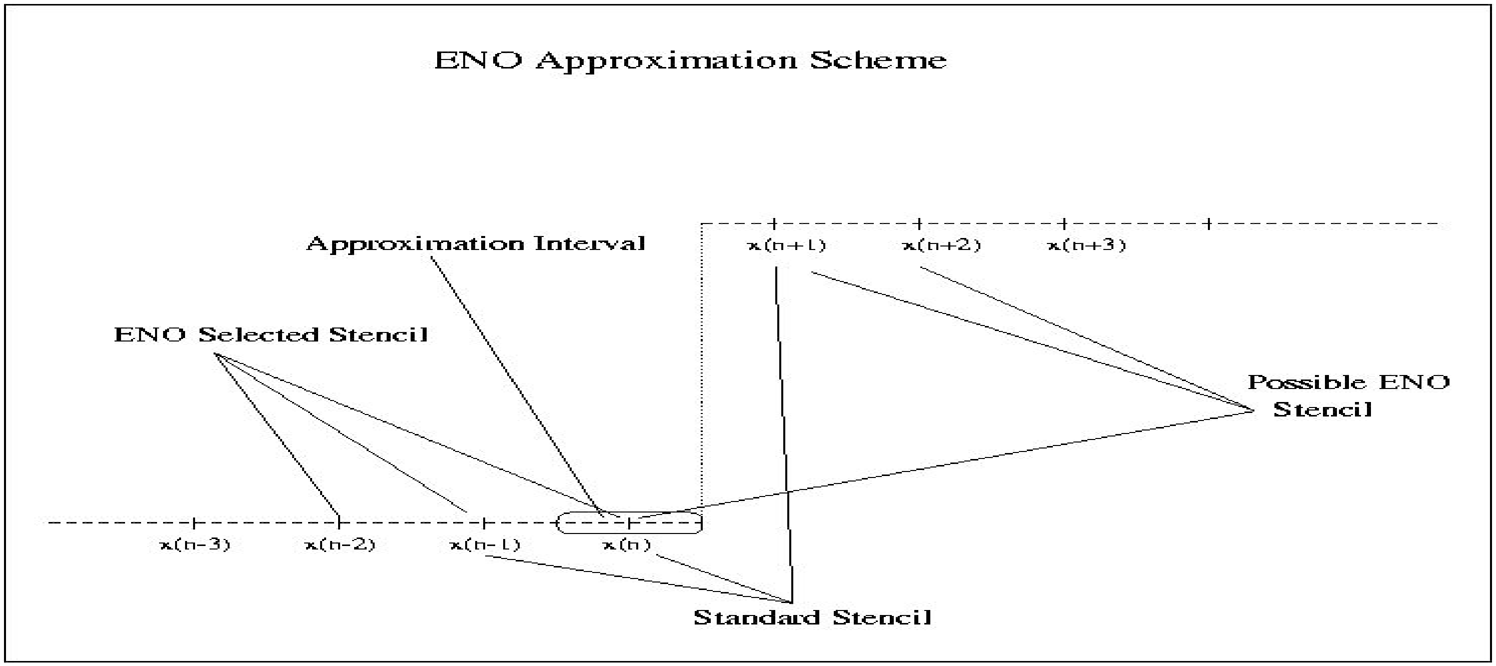
- ¥Lower the order of filter at discontinuities:
  - ¥Claypoole, Davis, Sweldens, Baraniuk[99]:Adaptive lifting
  - ¥Limitation: lower the order of accuracy
- ¥ENO one-sided approximation:
  - ¥Harten (93,94)
  - ¥At each point, adaptively form interpolation polynomial
  - ¥Never differencing crossing discontinuities.
  - ¥Limitation:
    - ¥Difficult for pyramidal wavelet transforms
    - ¥Need function values to form divided difference table at each point
    - ¥More extra cost
  - ¥Cohen and collaborators: some recent advances

Approaches ...

¥ENO-wavelets:

- ¥Do not want to change filters
- ¥Main idea: Change data adaptively
- ¥Make use of ENO s one-sided information idea
- ¥Goal: Do not generate large high frequency coef.

The ENO Idea



- ENO: invented by Harten, Engquist, Osher, Chakravarthy (87)
- Use one-sided information
- Newton divided differences to select smoothes stencil
- Very popular and successful in shock capturing and CFD

The ENO-wavelets Idea

¥Assume:

- ¥Know location of discontinuities
- ¥Discontinuity Separation Property (DSP):
  - ¥Two consecutive jump points separated by $l+3$ data points

¥Idea:

- ¥Use extrapolation from smooth side of jumps
- ¥Use same filters, but applied to smooth data

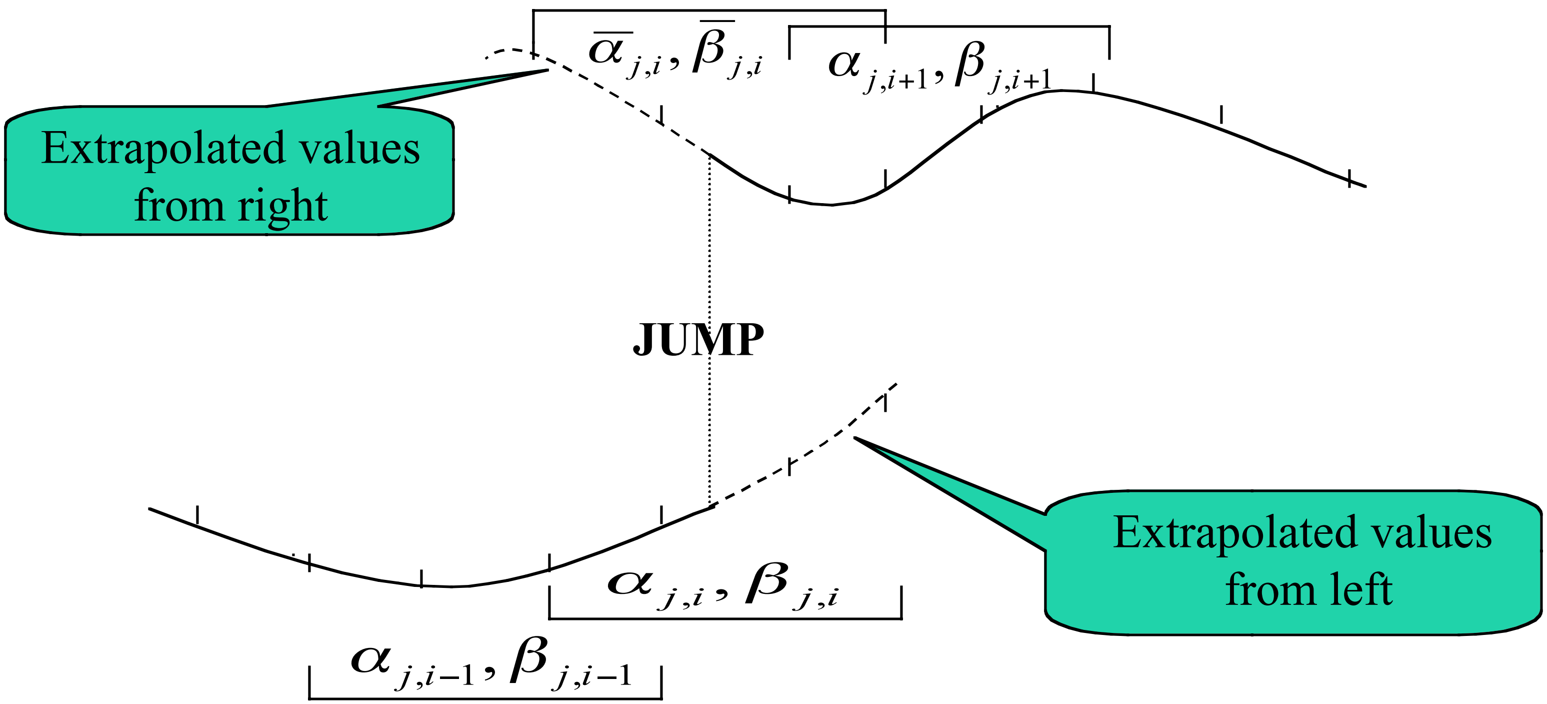
¥Must take care of

- ¥Retain accuracy order p
- ¥Minimal extra cost and storage
- ¥Invertibility: able to recover the original data

Direct Function Extrapolation

¥From left side $\longrightarrow$ Values $\alpha_{j+1,m}$

¥From right side $\longrightarrow$ Values $\overline{\alpha}_{j+1,m}$

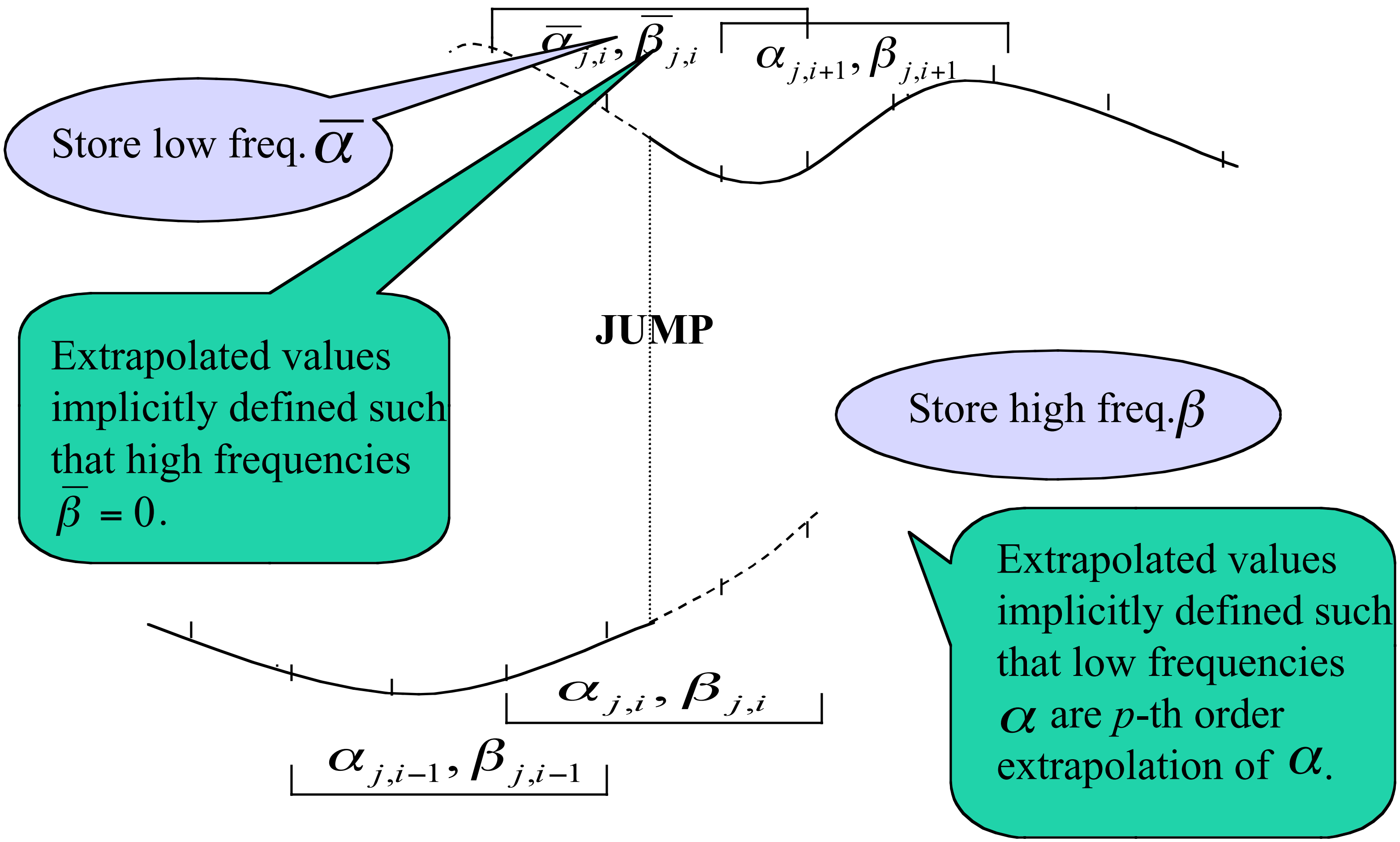


¥Apply same filters to smooth data on both sides

¥**Problem**: double storage at each discontinuity

Coarse Level Extrapolation

¥Extrapolate coarse level coefficients to determine the the fine level values.



Example 1

- Data: $\begin{pmatrix} 1 & 1 & 1 & 2 & 2 & 2 \end{pmatrix}$
- Standard Haar: $\alpha = \begin{pmatrix} \frac{2}{\sqrt{2}} & \frac{3}{\sqrt{2}} & \frac{4}{\sqrt{2}} \end{pmatrix}, \beta = \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & 0 \end{pmatrix}$
- Standard linear approximation:

$$\begin{pmatrix} 1 & 1 & \frac{3}{2} & \frac{3}{2} & 2 & 2 \end{pmatrix}$$

Do NOT store boxed values.

- ENO-Haar:

Extrapolation: $\begin{pmatrix} & & y & 2 & 2 & 2 \\ 1 & 1 & 1 & x & & \end{pmatrix}$

Coefficients: $\alpha = \begin{pmatrix} & \frac{4}{\sqrt{2}} & \frac{4}{\sqrt{2}} \\ \frac{2}{\sqrt{2}} & \boxed{\frac{2}{\sqrt{2}}} & \end{pmatrix}, \beta = \begin{pmatrix} & \boxed{0} & 0 \\ 0 & 0 & \end{pmatrix}$

Store:

$$\alpha = \begin{pmatrix} \frac{2}{\sqrt{2}} & \frac{4}{\sqrt{2}} & \frac{4}{\sqrt{2}} \end{pmatrix}, \beta = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$$

Linear approximation: same as the initial

Example 2

¥Data: $\begin{pmatrix} 0 & 1 & 2 & 10 & 11 & 12 \end{pmatrix}$

¥Standard Haar: $\alpha = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{12}{\sqrt{2}} & \frac{23}{\sqrt{2}} \end{pmatrix}, \beta = \begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{8}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$

¥Standard linear approximation:

$$\begin{pmatrix} 0.5 & 0.5 & 6 & 6 & 11.5 & 11.5 \end{pmatrix}$$

Do NOT store boxed values.

¥ENO-Haar:

¥Extrapolation $\begin{pmatrix} & & & y & 10 & 11 & 12 \\ 0 & 1 & 2 & x & & & \end{pmatrix}$

¥Coefficients:

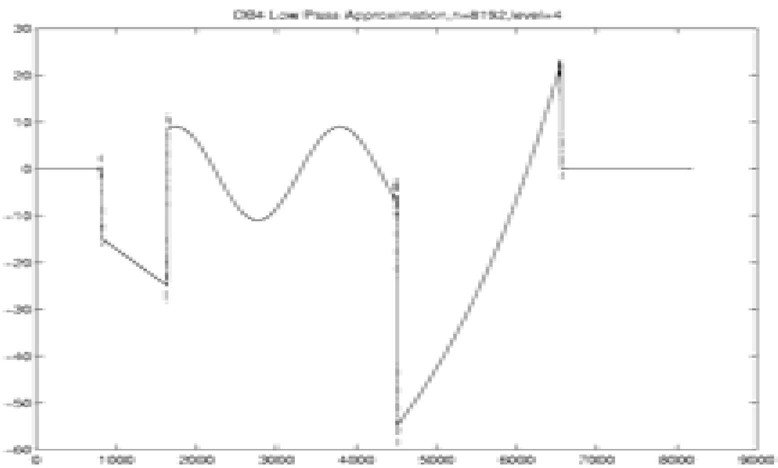
$$\alpha = \begin{pmatrix} & \frac{20}{\sqrt{2}} & \frac{23}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \boxed{\frac{1}{\sqrt{2}}} & \end{pmatrix}, \beta = \begin{pmatrix} & & \boxed{0} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{3}{\sqrt{2}} & & \end{pmatrix}$$

¥Store:

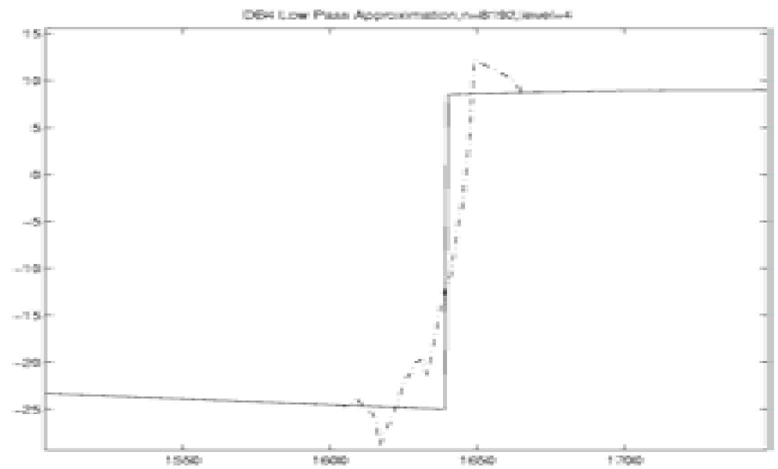
$$\alpha = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{20}{\sqrt{2}} & \frac{23}{\sqrt{2}} \end{pmatrix}, \beta = \begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{3}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

¥Linear approximation: $\begin{pmatrix} 0.5 & 0.5 & 0.5 & 10 & 11.5 & 11.5 \end{pmatrix}$

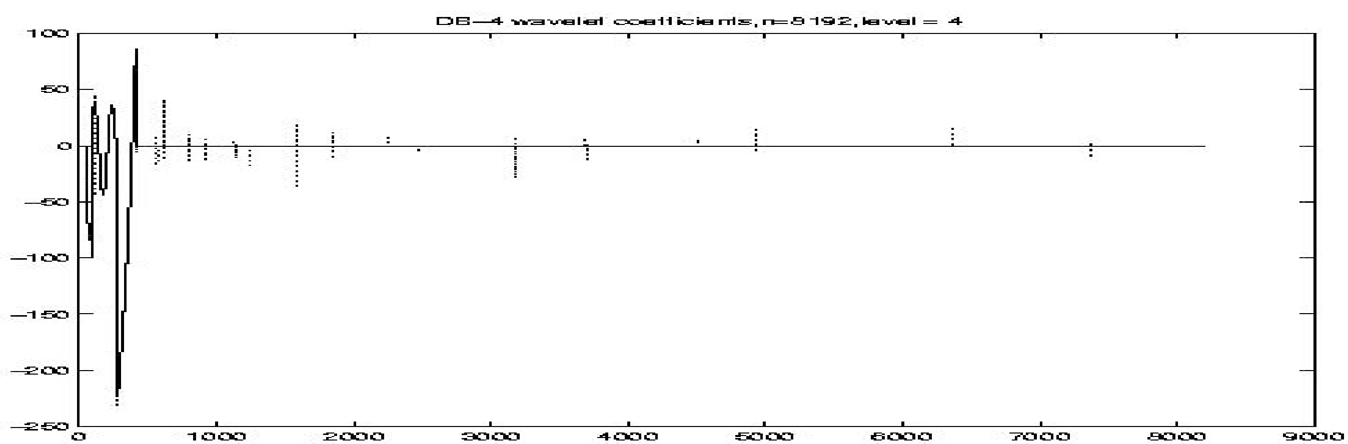
Example of ENO-DB4



4-level DB4 v.s. ENO-DB4



Zoom in at a discontinuity



Large high frequencies in DB4 but not in ENO-DB4

Error Bound and Stability

Error: $f_j(x)$ is the j-th level ENO-wavelet approximation. If $f_{j-1}(x)$ satisfies the DSP, then

$$\left\| f(x) - f_j(x) \right\| \le C(\Delta x)^p \left\| f^{(p)}(x) \right\|_{(a,b)\setminus D}$$

¥Denote $\Delta x = 2^{-j}$

¥D is the set of discontinuities

¥Wavelet function has p vanishing moments

¥The standard error bound depends on $\left\| f^{(p)}(x) \right\|_{(a,b)}$

Stability: If $\| f(x) - g(x) \| \le \varepsilon$ and same set of discontinuities detected, then

$$\| f - g \| \le O(\varepsilon)$$

Outline of the proof

¥Consider individual jump

¥Consider three cases

¥Direct function extrapolation: preserve order

¥Extend $\beta = 0 \longrightarrow$ *(p-1)*-th order smooth extension

¥Extrapolating α's : extrapolation in wavelet spaces $\longrightarrow$ same order extrapolation in function space

Properties

¥Output sequence: same size as input sequence

Half high frequencies and half low frequencies.

¥Perfectly invertible

¥Extra storage: remember the location of jumps (ENO mapping)

¥Cost: Algorithmic complexity remains $O(n)$

¥Standard cost: $O(nl)$ ¥Extra cost: $O(dl)$, d: number of jumps

¥Ratio of extra over standard: $O(d/n)$

¥Keep p-th order accuracy

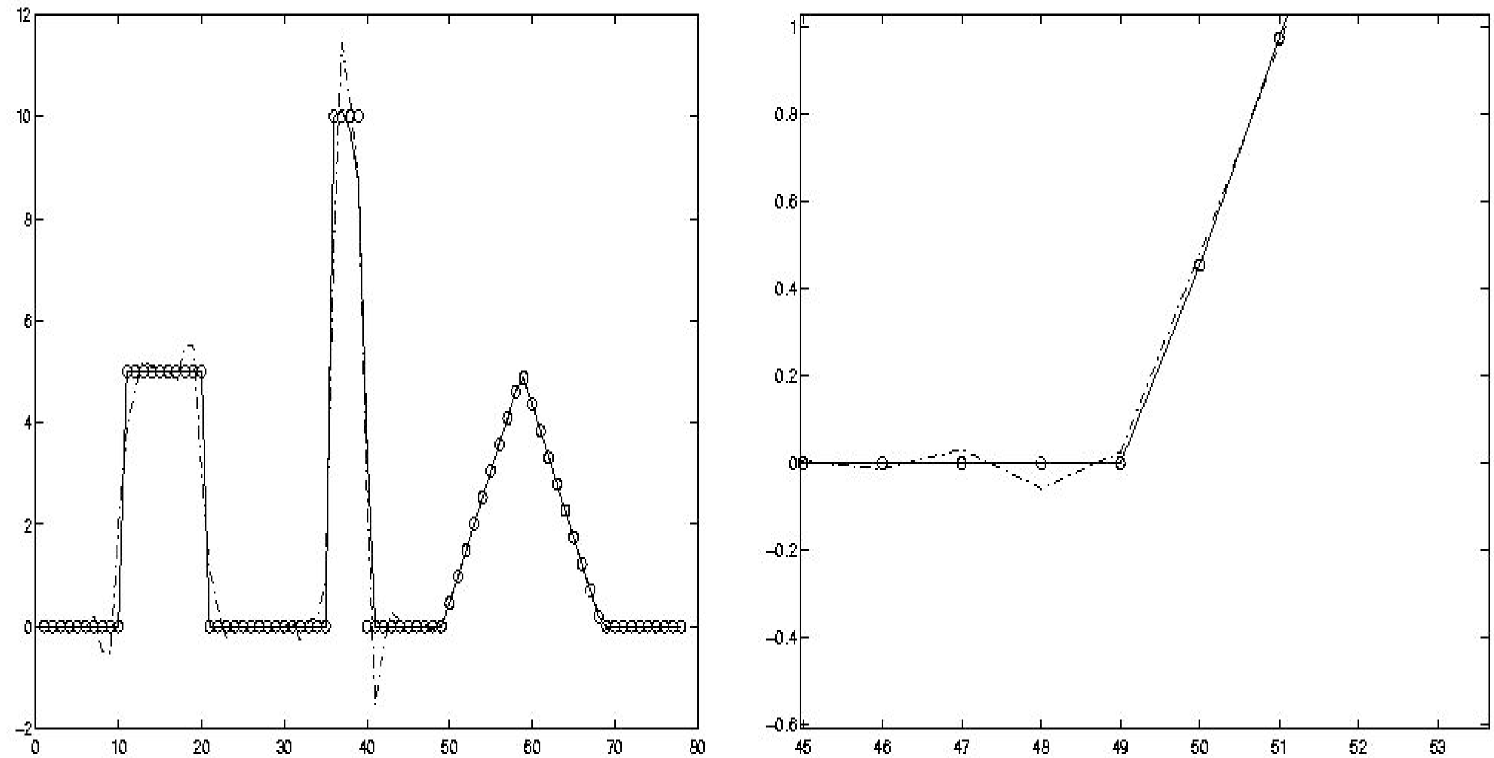
¥Stable

¥Can use other extrapolation schemes

¥Apply to other (non-orthogonal) wavelets

¥2-D by tensor products

Tests on DSP



The level-1 ENO-DB6 v.s. DB6 at places where

¥DSP satisfied (left bump): exactly.

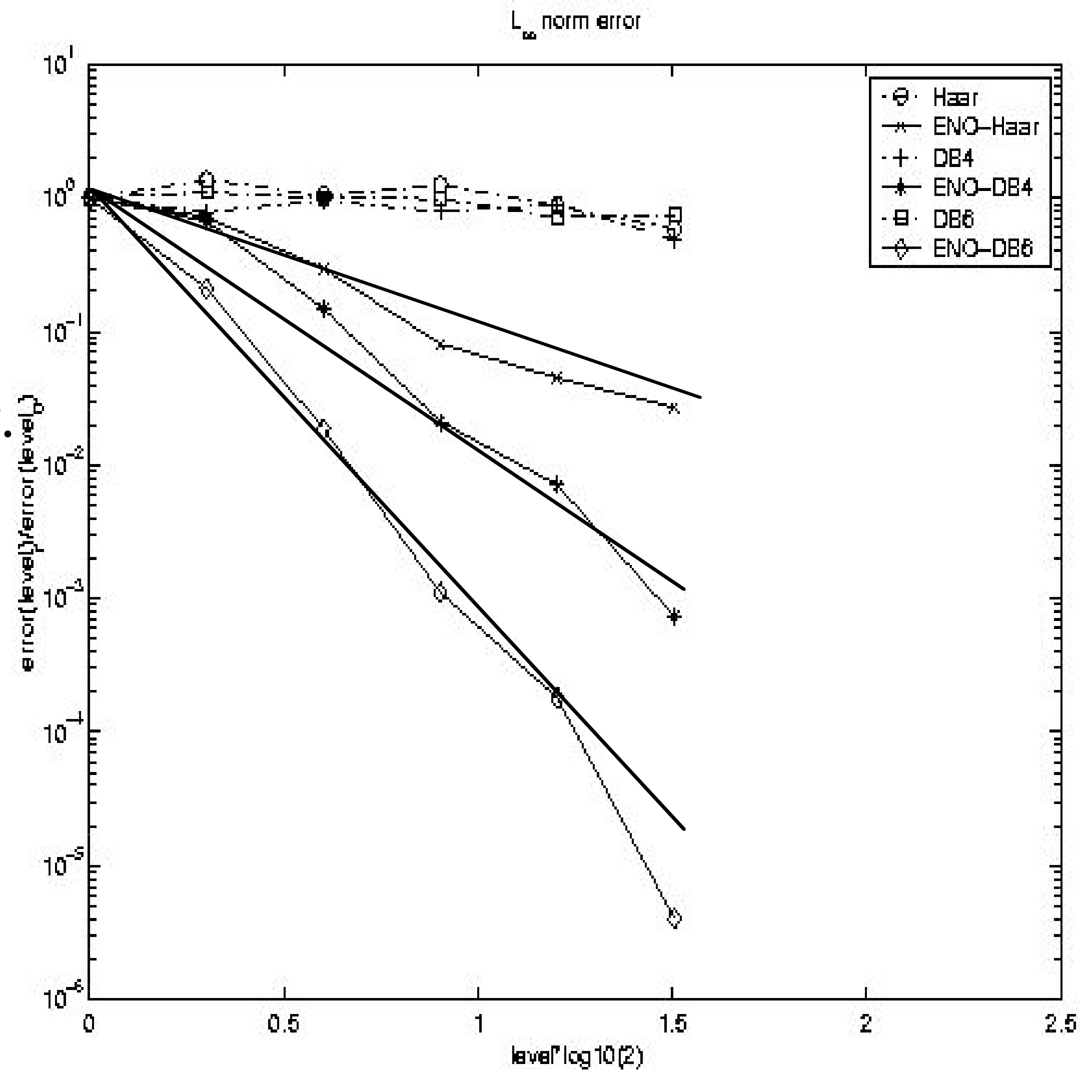
¥DSP invalid (middle bump): error comparable.

¥Jump in derivatives (right corners): exactly.

Order of Approximation

Order in L^∞

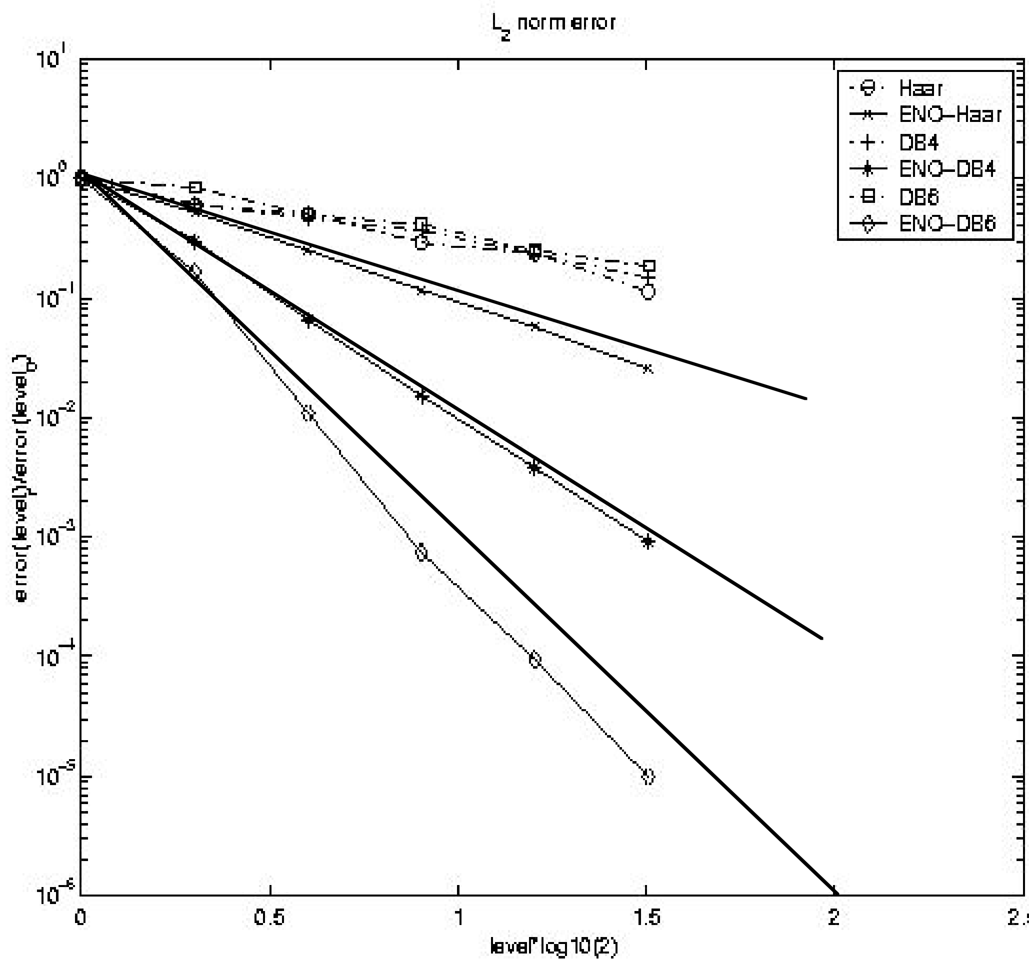
¥All standard: No order.
¥ENO-Haar: first.
¥ENO-DB4: second.
¥ENO-DB6: third.



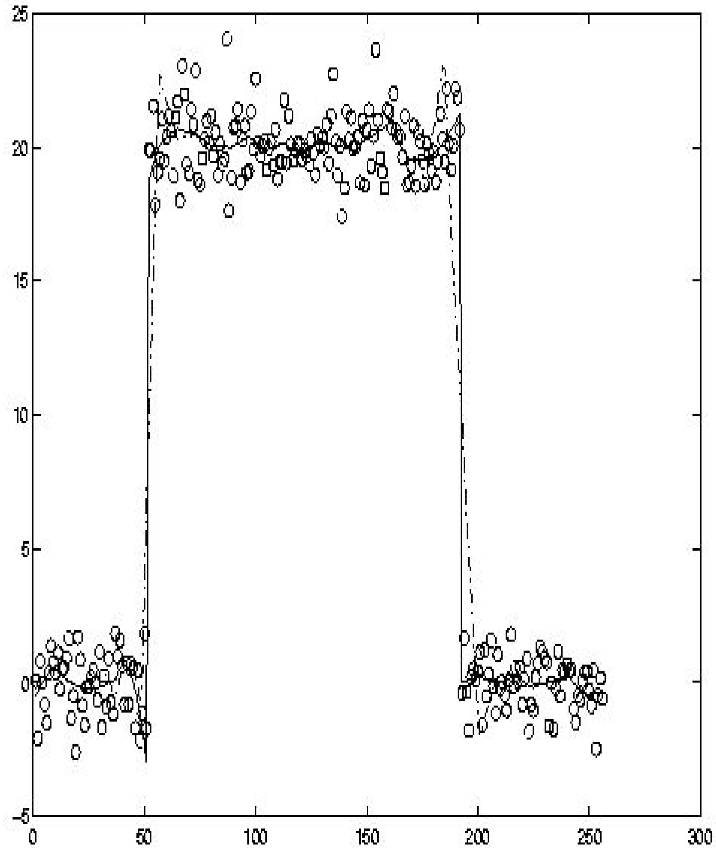
Order of Approximation ...

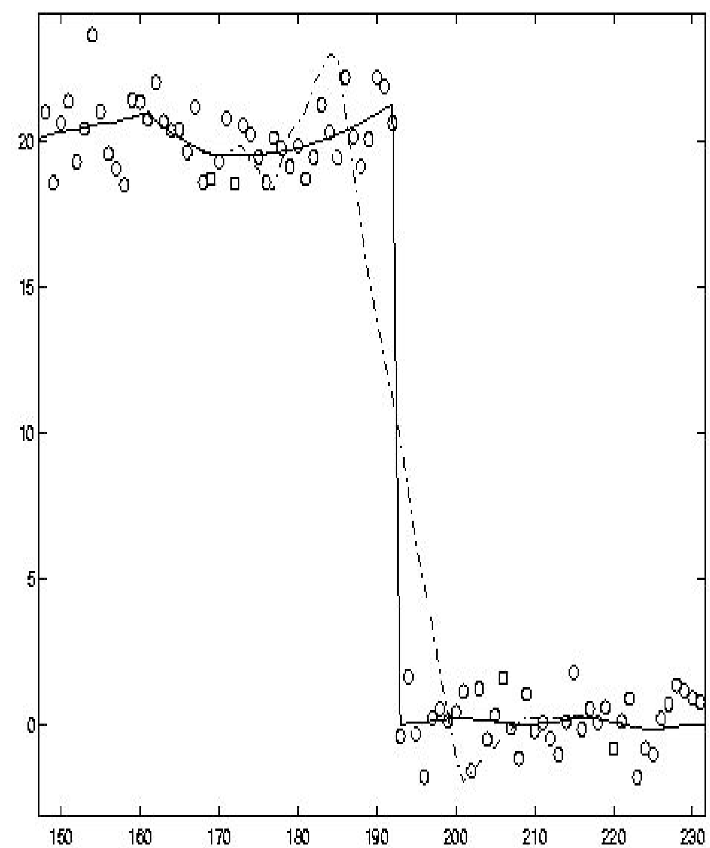
Order in L^2

¥All standard: No order
¥ENO-Haar: first
¥ENO-DB4: second
¥ENO-DB6: third



Noisy Data



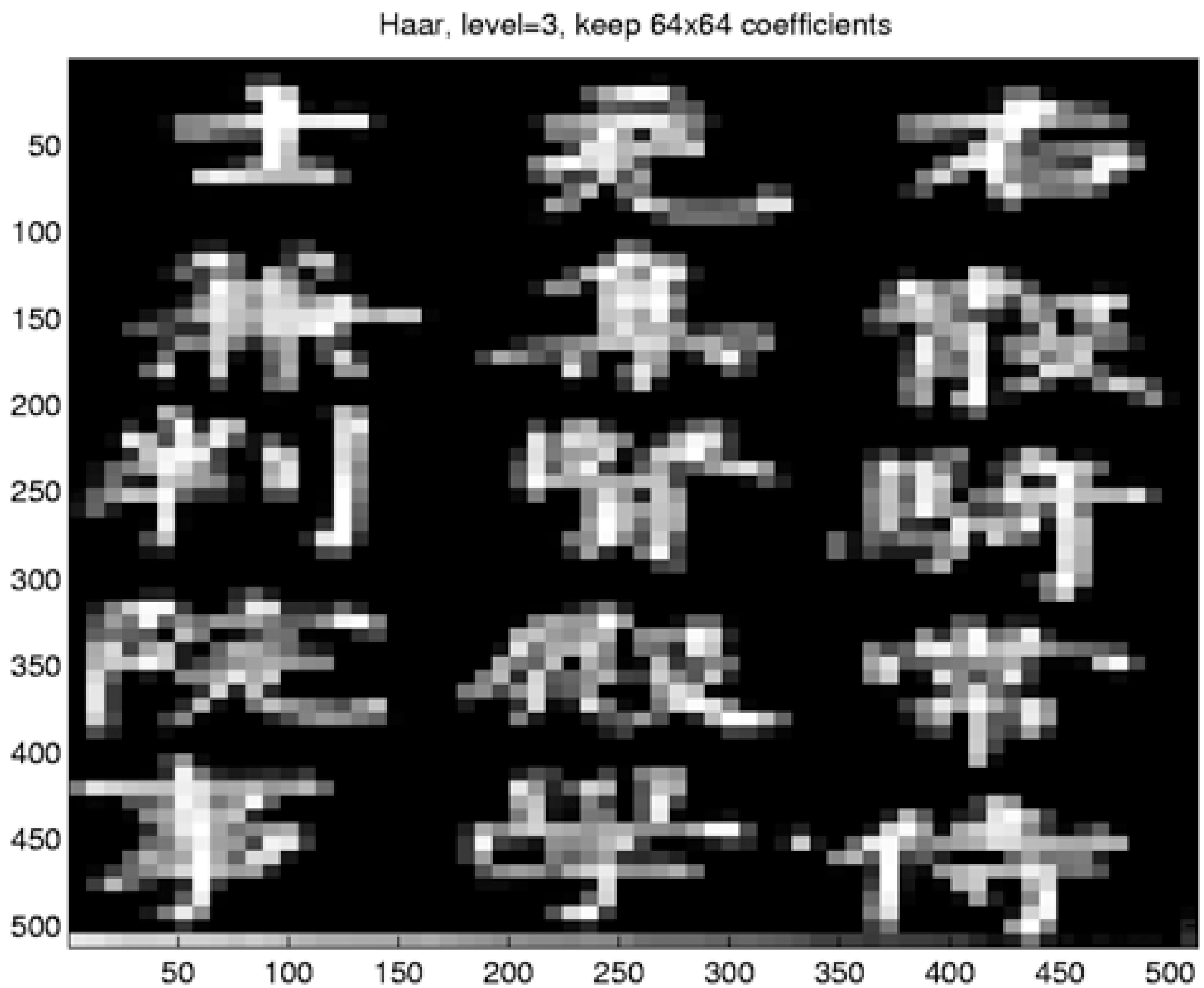


3-level ENO-DB6 v.s. DB6

2-D Example

Original 2-D Function

Haar

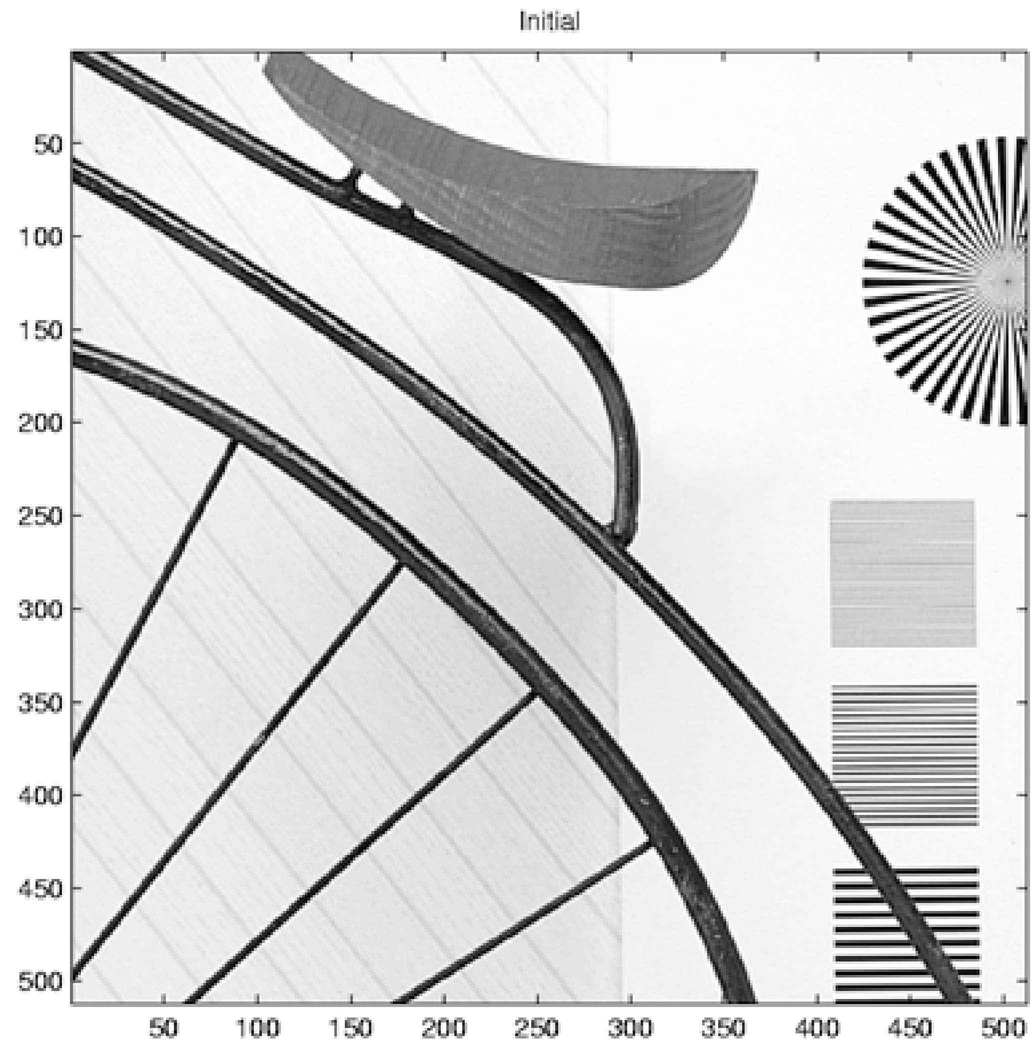


3-level Haar, edges and interior are fuzzy

ENO-Haar

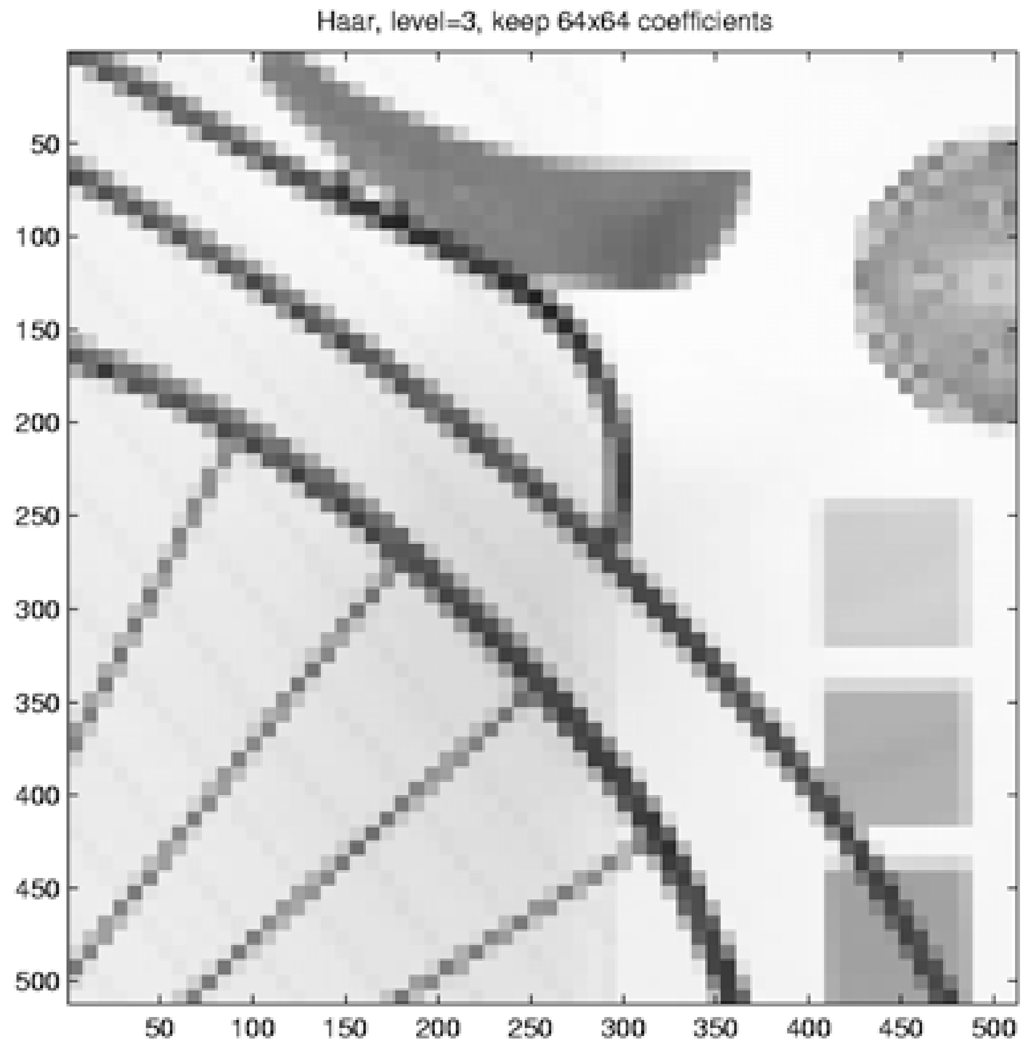
3-level ENO-Haar, Edges and interior are clearer

2-D Example ...



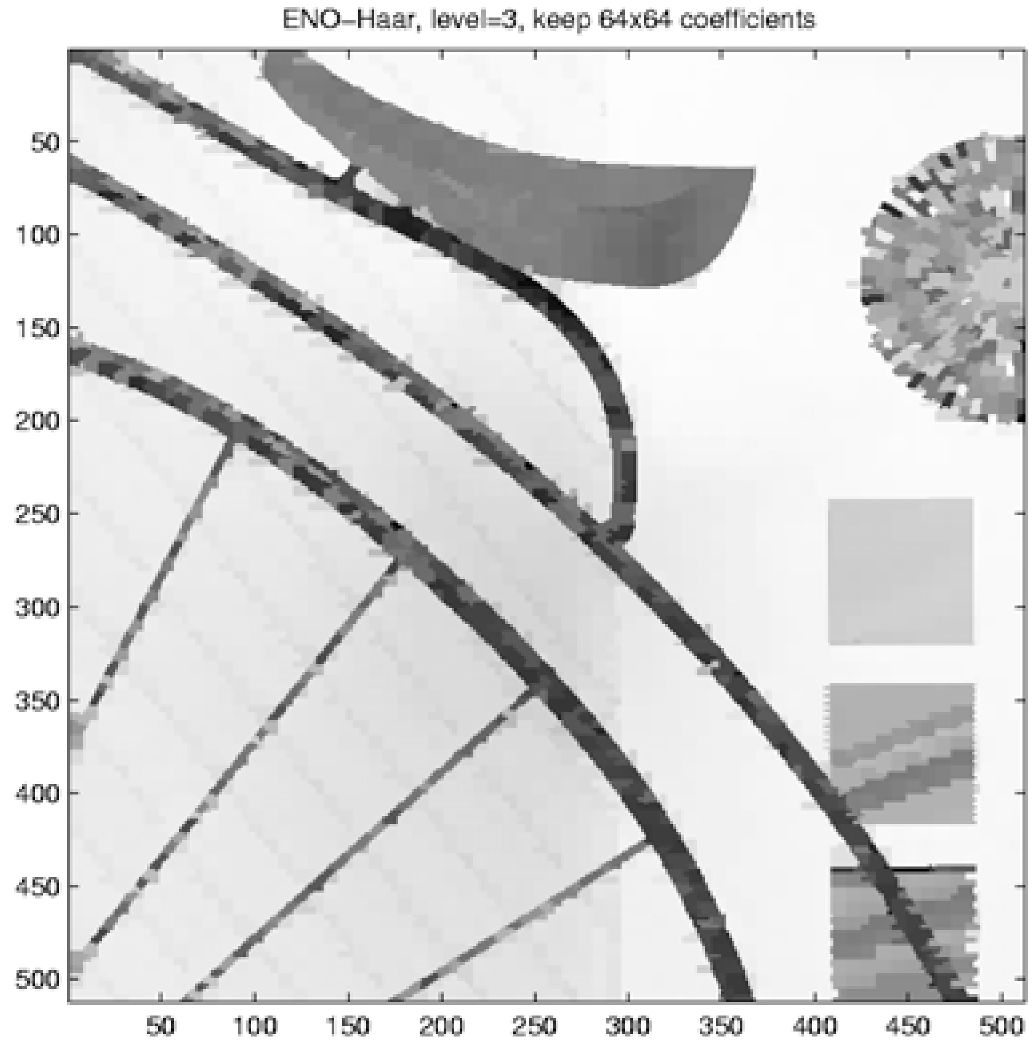
Original 2-D Function

Haar



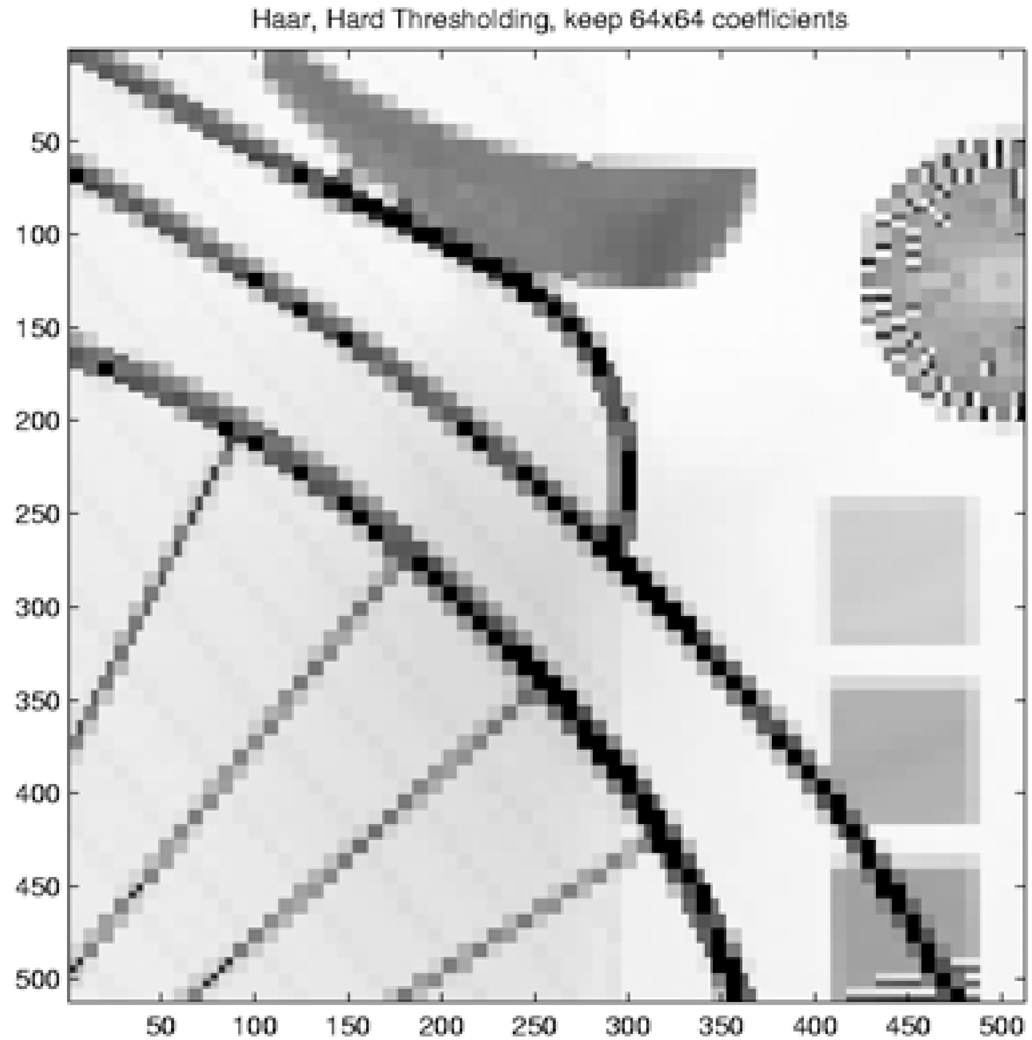
3-level Haar, keep 64x64 coefficients

ENO-Haar



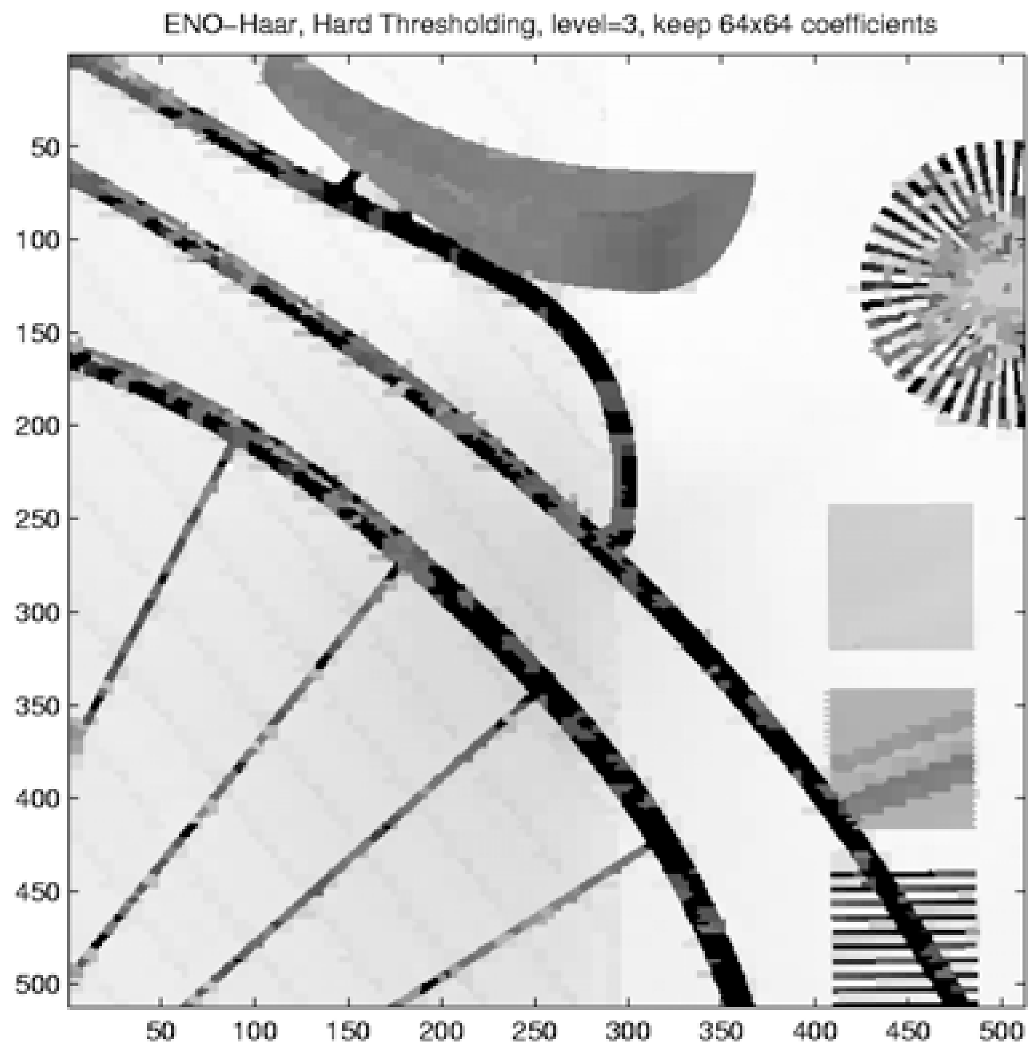
3-level ENO-Haar, keep 64x64 coefficients

Haar, Hard Thresholding



3-level Haar, Hard thresholding, keep 64x64 coef.

ENO-Haar, Hard Thresholding



3-level ENO-Haar, Hard thresholding, keep 64x64 coef.

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Application in Image Compression

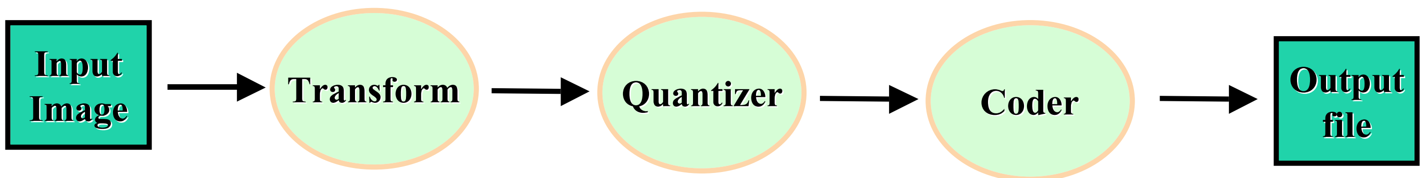
¥Represent images by fewer wavelet or ENO-wavelet coefficients

¥Is this sufficient for the efficiency of image compression?

¥Answer is NO

¥Reason: there are more components, not only transforms, in a compression system, and they have to be considered too.

Components of Image Compression Systems



¥ **Transform: redundancy removal,**
e.g. DCT, Wavelets

¥ **Quantizer: entropy (information) reduction,**
e.g. Scalar quantizer: real number → ***integers***

¥ **Coder: lossless coding**
e.g. Huffman, LZW, arithmetic coding

Task

¥ Efficiency in storage

Rate (bits/pixel) as low as possible.

¥ Accuracy in representations

Distortion (error: PSNR) as small as possible.

¥ Optimize rate-distortion trade-off on a range of rates specified by the users.

Multi-resolution (MR) Codes

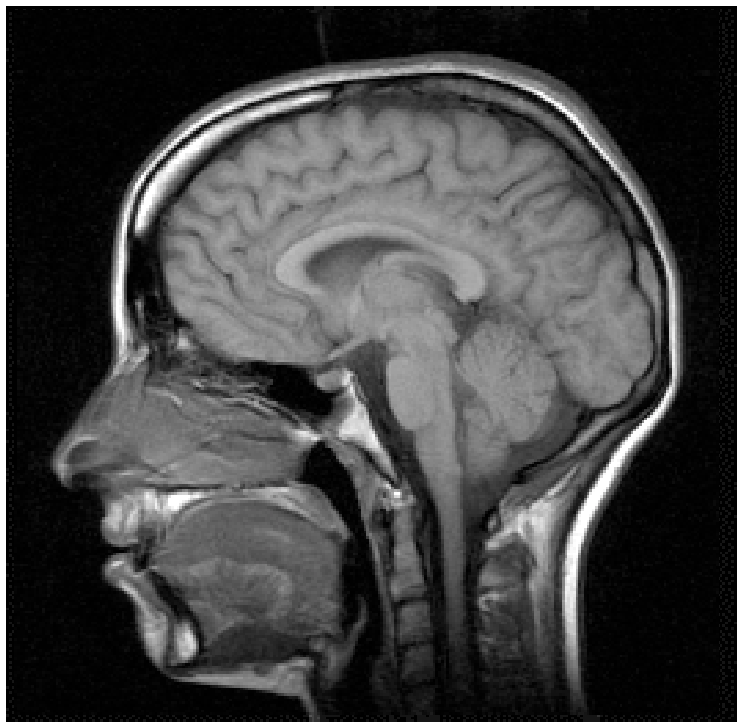
¥MR code: a single compression system to reproduce at a variety of rates and resolutions.

¥Also called progressive transmission, embedded or successive refinement codes.

¥Low-resolution are embedded in higher-resolution of the same data set.

¥Applications: a single source must be accessible to different users or at different rates that varies, e.g. images on internet.

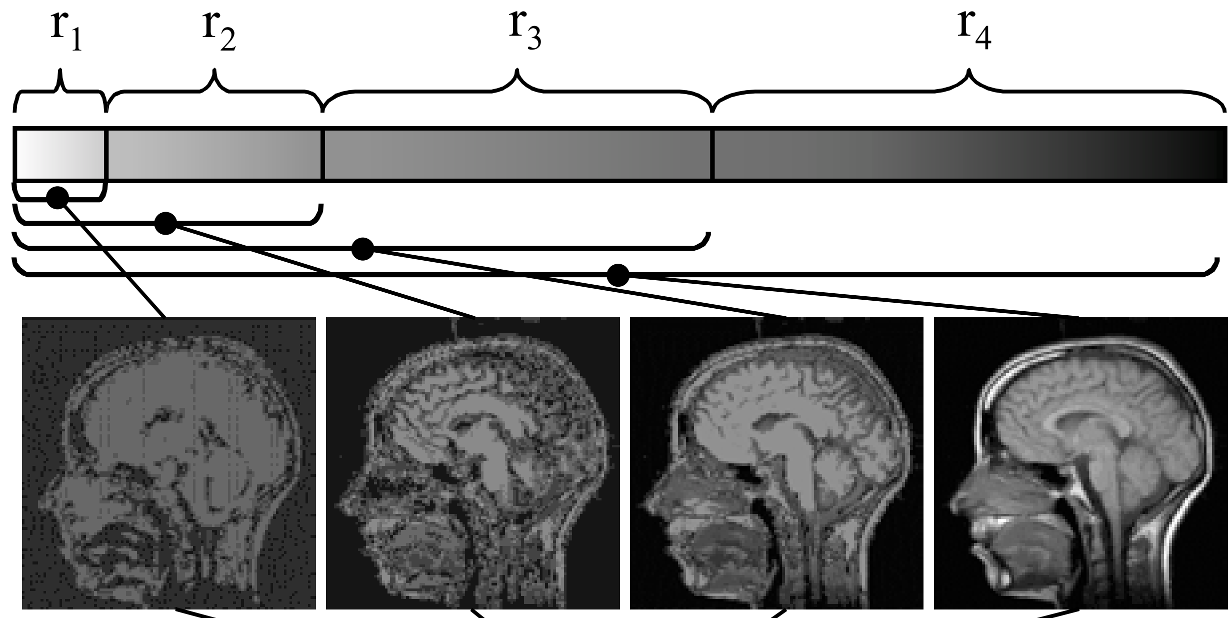
Multi-resolution (MR) Codes

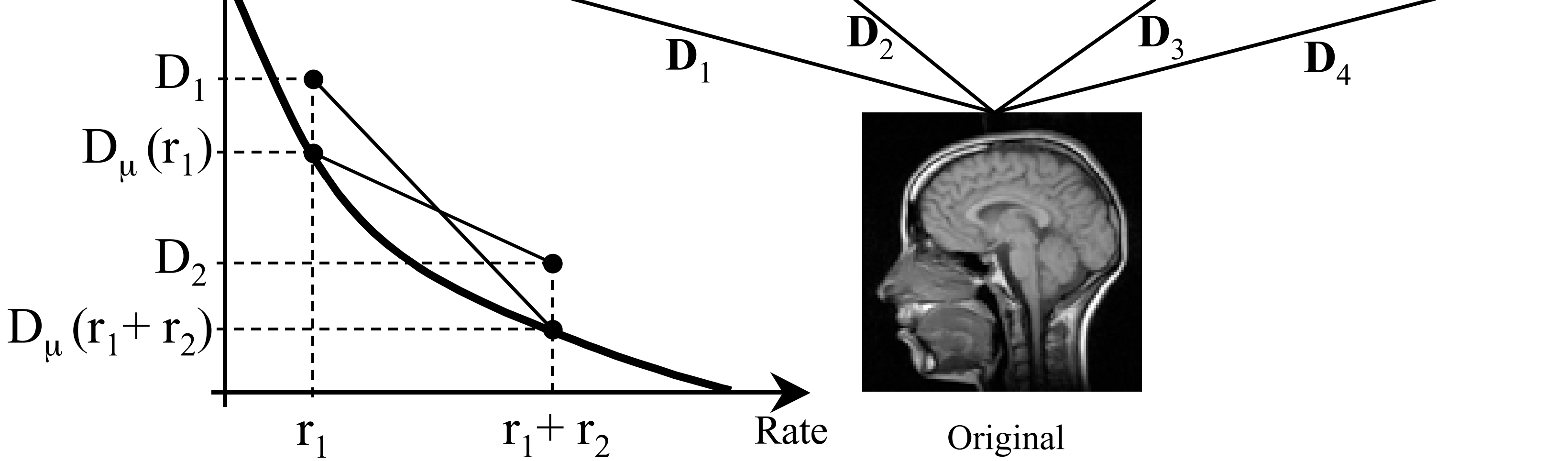


Multi-resolution (MR) Codes

¥Embedded Coding

¥Rate-Distortion Trade-off





Why MR Codes?

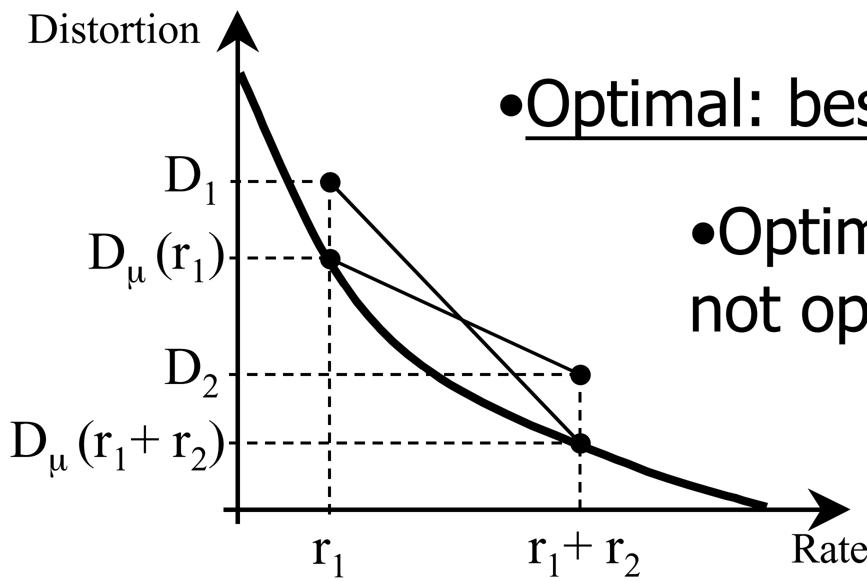
¥Applications: a single source must be accessible to different users or at different rates that varies, e.g. images on internet.

¥Different demands at different rates

Rate-Distortion Trade-off

- A lower bound on the rate-distortion curves (Shannon).

- One can design codes to achieve the bound arbitrarily close at a given rate.



- <u>Optimal: best rate-distortion trade-off.</u>
  - Optimal code at one rate is not optimal at a different rate

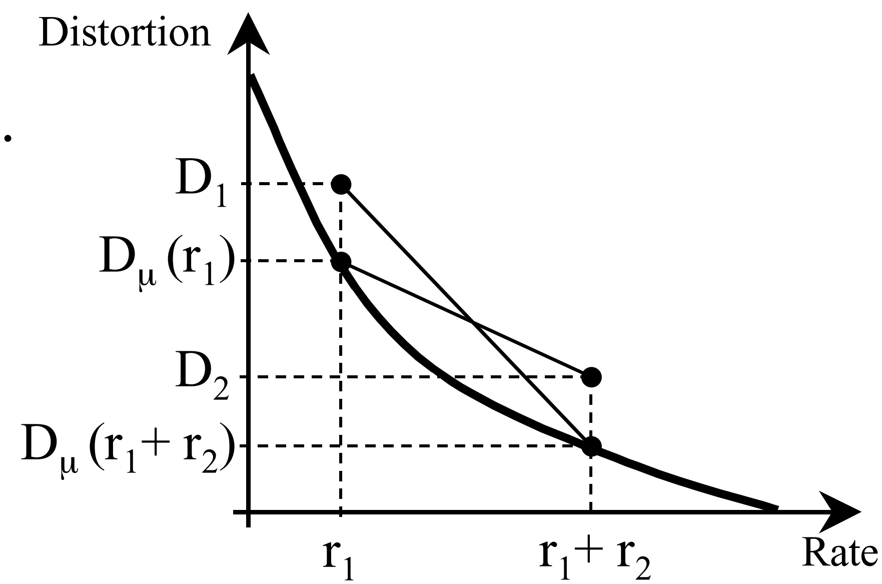
¥Optimize according to the rate priorities

Multi-resolution (MR) Codes

¥Creating an L-resolution code with optimal performance at ***ONE*** of its L resolutions: **Not Difficult**

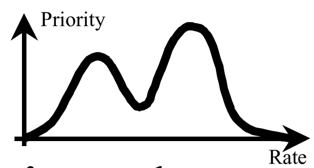
¥Constructing an L-resolution code with optimal performance at ***MORE THAN ONE*** of its L resolutions **may not be possible**

¥Priorities may be necessary.



Rate-Distortion Optimization

- MR Lagrangian measure $J = \sum_{\ell=1}^{L} \left[\alpha_\ell D_\ell + \beta_\ell R_\ell \right]$
- Priorities $\{\alpha_\ell, \beta_\ell\}$



- Minimize $J \longrightarrow$ optimal performance
- Trade-off of ENO-Wavelet: Storage of locations of discontinuities v.s. Relative savings of smaller high freq.

State of the art compression: GTW

¥ Group Testing on Wavelet (GTW) coefficients is a recent (Hong & Ladner 2000) *lossy* coding technique which can efficiently represent few significant elements in a large pool of coefficients.

¥ Zero-tree type of coding algorithm implemented in bit-plane fashion: instead of deciding whether to keep a whole coefficient, the decision is made on every bit of the coefficient.

¥ Key trade-off: for every-one bit of a coefficient, storing it as 1 will decrease the distortion, but increase the rate.

State of the art compression: GTW

¥Hong & Ladner, 2000

¥Zero-tree type of bit-plane coding

¥Use Group Testing (GT) to wavelet coefficients

¥GT: an efficient way to identify few significant elements in a large pool

Optimization of GTW

¥Dugatkin, Zhou, Chan and Effros (2002)

¥Optimize the Lagrangian rate-distortion trade-off performance: $J = \sum_{\ell=1}^{L} \left[\alpha_\ell D_\ell + \beta_\ell R_\ell \right]$, α, β are weights.

¥Incorporate ENO-Wavelet Coefficient in the optimization procedure.

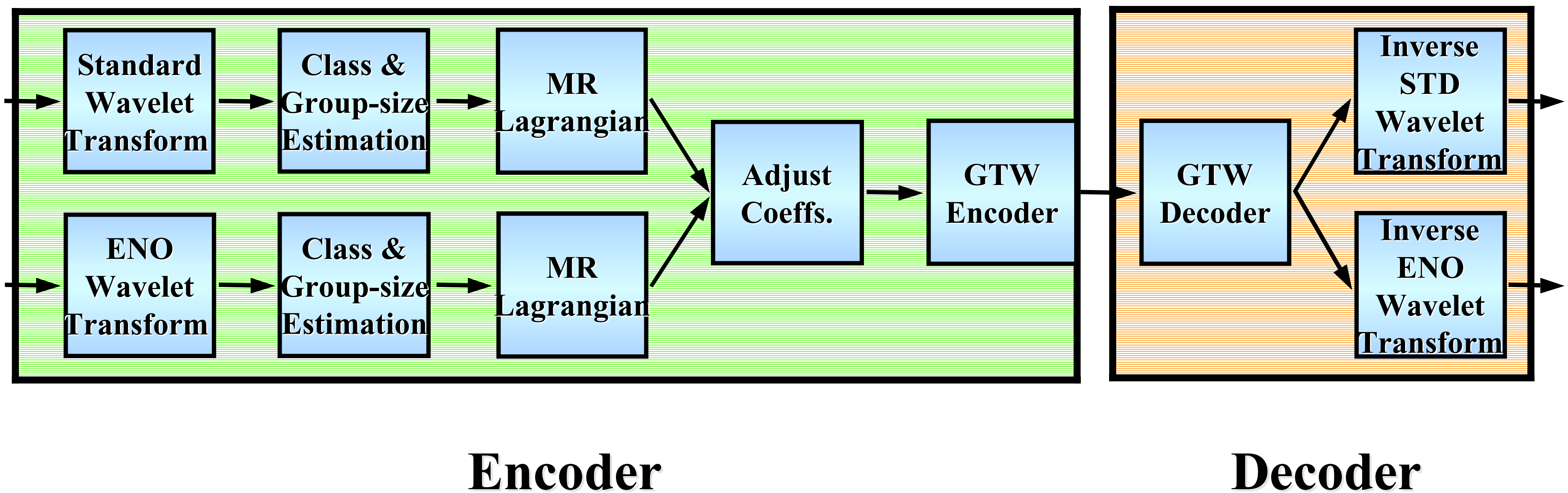
¥Trade-off of ENO-Wavelet: Storage of locations of discontinuities v.s. Relative savings of smaller high freq., which is considered in the Lagrangian

Optimization of GTW

¥Optimize the Lagrangian rate-distortion trade-off performance at each bit-plane

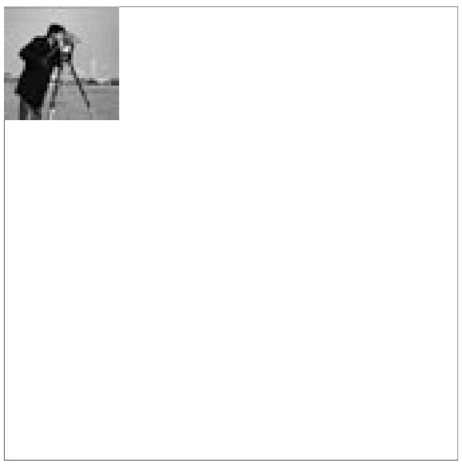
¥Incorporate ENO-Wavelet Coefficient in the optimization procedure.

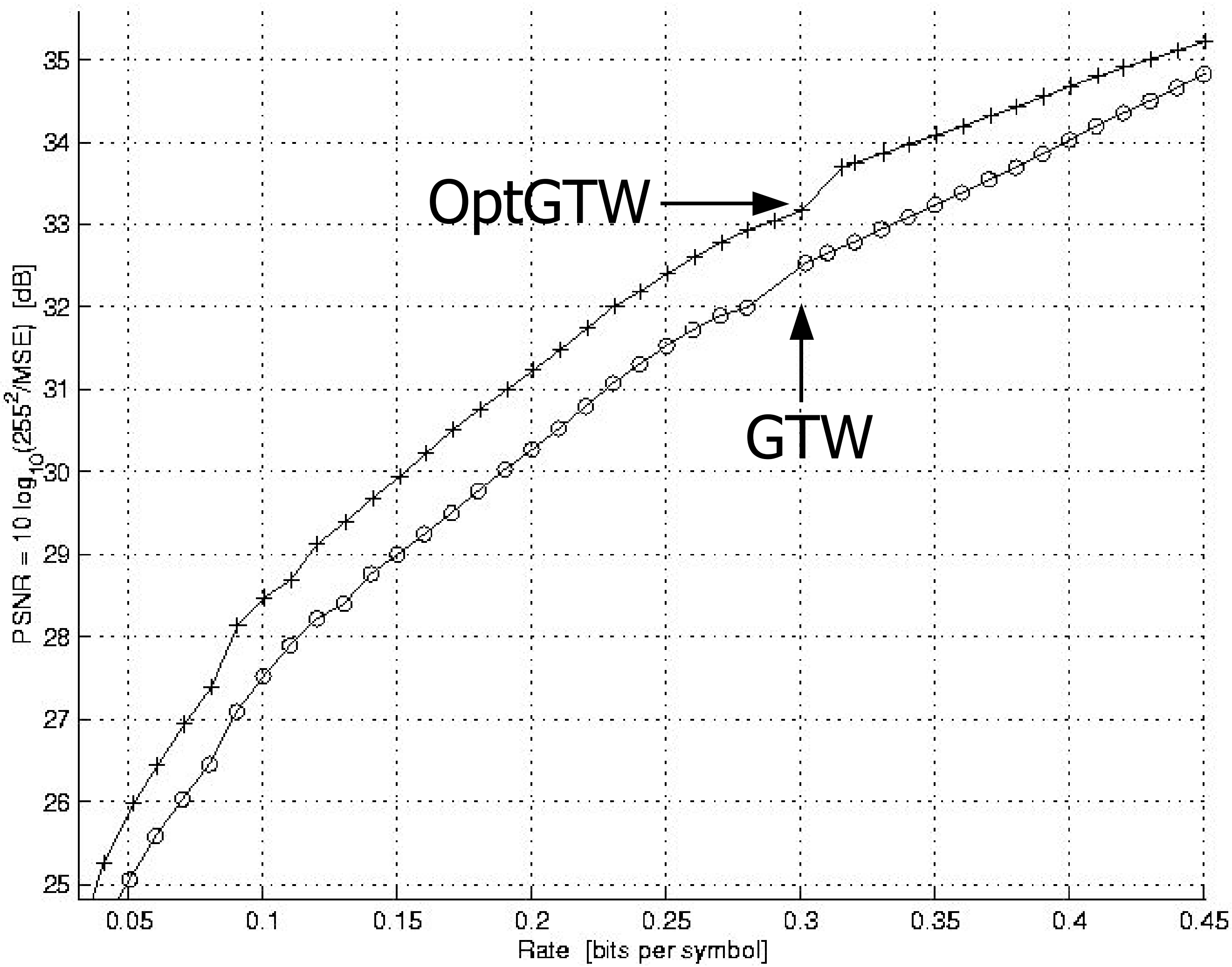
Algorithm



Standard Wavelet Transform
Class & Group-size Estimation
MR Lagrangian
ENO Wavelet Transform
Class & Group-size Estimation
MR Lagrangian
Adjust Coeffs.
GTW Encoder
GTW Decoder
Inverse STD Wavelet Transform
Inverse ENO Wavelet Transform
Encoder
Decoder

PSNR vs. Rate results

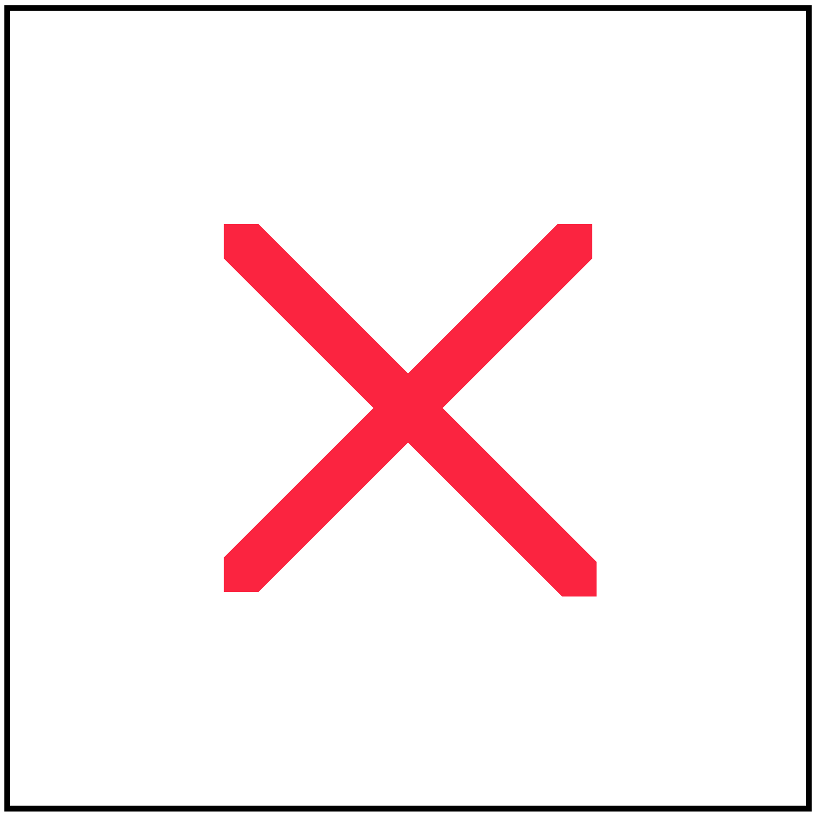




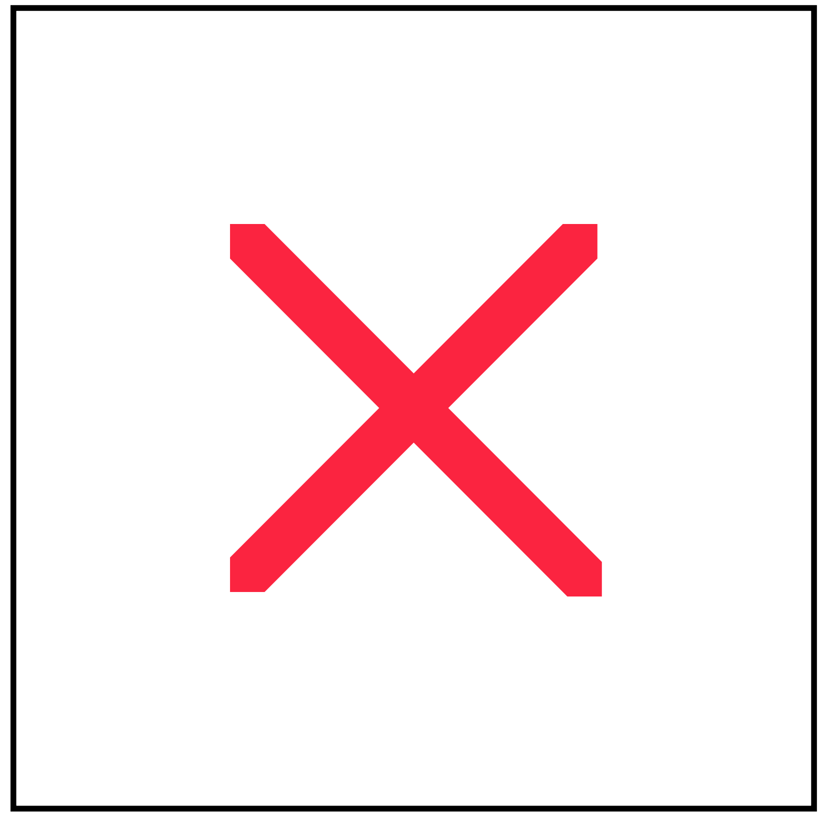
¥Save over 1.0dB

Visual quality

Standard GTW



New OPT-GTW with ENO



cameraman reconstructed image at R=0.1bpp, better edge reconstruction

Outline

¥Introduction & Motivations

¥ENO-Wavelet Transforms

¥Application in Image Compression

¥Total Variation (TV) Model for Wavelet Thresholding

¥Conclusion

Topic 2

TV models for wavelet thresholding and its application in image compression and denoising

TV Image Processing

¥Great success of TV in image processing, (Rudin, Osher, Fatemi): smooth oscillations but retain sharp edges.

¥Many people work on it: Chan, Osher s group, Vogel, Santosa, Dodson, Plemmons, Chambolle, Lions

¥Applications in image processing: Denoising, Debluring, Segmentation, Color images, Image on manifolds, Digital TV filters

Motivations for Topic 2

¥Great success of TV in image processing, (Rudin, Osher, Fatemi): smooth out oscillations but retain sharp edges.

¥Denoising: (Chan, Osher s group, Vogel ...)

¥Blind deconvolution: (Chan-Wong)

¥Color TV:(Chan-Blomgren)

¥Digital TV filters: (Chan-Shen)

¥Segmentation:(Chan-Vese)

¥Images on manifolds: (Osher-Cheng)

TV in Image Processing

¥Chambolle, DeVore, Lee and Lucier: min. in Besov by wavelets

¥TV denoising + Wavelet Thresholding: better ratio or quality (Chan-Zhou, 1998)

¥Oscillations generated by thresholding increase TV norm

TV in Wavelet Thresholding

¥ Chan & Zhou (2000): TV optimized wavelet coefficients in image compression and denoising.

¥ Durand & Froment (2001): fixed the retained wavelet coefficients in hard thresholding and adjust others to minimize the TV norm to erase the oscillations.

¥ Candes (2001): TV post processing for curvelet thresholding

General TV Model

$$\min_{\beta_{j,k},(j,k)\in I} \lambda \int \left|\nabla u(\beta, x)\right| dx + \left\|u - z\right\|_2^2$$

S.T. $\quad |I| = m$

Where

$$u = \sum_{j,k} \beta_{j,k} \varphi_{j,k}(x)$$

$$z = \sum_{j,k} \alpha_{j,k} \varphi_{j,k}(x)$$ – Observed image

m ——Given integer

General TV Model

¥Nonlinear integer optimization

¥Difficulties: Integer constraint, too many local solutions, nonlinear equations.

¥Selection of λ : L-curve, training images

General TV Model

$\lambda \longrightarrow 0$, Standard Thresholding

$\lambda \longrightarrow \infty$, constants

λ : Control the small feature size to preserve (Strang-Chan).

TV Hard Thresh. Model

$$\min_{\beta_{j,k},(j,k)\in I_H} \lambda \int \left|\nabla u(\beta, x)\right| dx + \|u - z\|_2^2$$

Euler-Lagrangian

$$-\lambda \int \nabla \left(\frac{\nabla u}{|\nabla u|} \right) \varphi_{j,k} dx + 2(\beta_{j,k} - \alpha_{j,k}) = 0$$

$$(j,k) \in I_H$$

Approximation to Constraint

$$|I| = m$$

Approximate by:

$$\Big(\sum_{(j,k)} \log(1+\beta_{j,k}{}^{2}) - m\Big)^2 \le \gamma^2$$ Olshausen & Field

$$\Big(\sum_{j,k} \left|\beta_{j,k}\right|^{p} - m\Big)^2 \le \gamma^2, p \to 0$$ Donoho(99)

TV Relaxation Models

$$\min_{\beta_{j,k},(j,k)\in I_H} \lambda\int\left|\nabla u(\beta,x)\right|dx + \left\|u-z\right\|_2^2 + \tau\Big(\sum_{j,k}\log(1+\beta_{j,k}{}^2) - m\Big)^2$$

OR

$$\min_{\beta_{j,k},(j,k)\in I_H} \lambda\int\left|\nabla u(\beta,x)\right|dx + \left\|u-z\right\|_2^2 + \tau\Big(\sum_{j,k}\left|\beta_{j,k}\right|^p - m\Big)^2$$

Euler-Lagrangians

$$-\lambda\int\nabla\left(\frac{\nabla u}{|\nabla u|}\right)\varphi_{j,k}dx+2(\beta_{j,k}-\alpha_{j,k})+2\tau\Big(\sum_{j,k}\log(1+\beta_{j,k}^2)-m\Big)\frac{\beta_{j,k}}{1+\beta_{j,k}^2}=0$$

Or

$$-\lambda\int\nabla\left(\frac{\nabla u}{|\nabla u|}\right)\varphi_{j,k}dx+2(\beta_{j,k}-\alpha_{j,k})+2\tau\Big(\sum_{j,k}|\beta_{j,k}|^p-m\Big)\frac{\beta_{j,k}}{|\beta_{j,k}|^{2-p}}=0$$

Numerics

¥Time Marching, Fixed-point, Primal-Dual...

¥Regularizations to prevent blow-up

¥Transform Data between wavelet spaces and physical space.

Fixed-point Iterations

Linearize the nonlinear terms by using previous approximations: e.g.

$$-\lambda \int \nabla \cdot \left(\frac{\nabla u^{n+1}}{\left|\nabla u^{n}\right|} \right) \varphi_{j,k} \, dx + 2(\beta_{j,k}{}^{n+1} - \alpha_{j,k}) = 0$$

$$(j,k) \in I_H$$

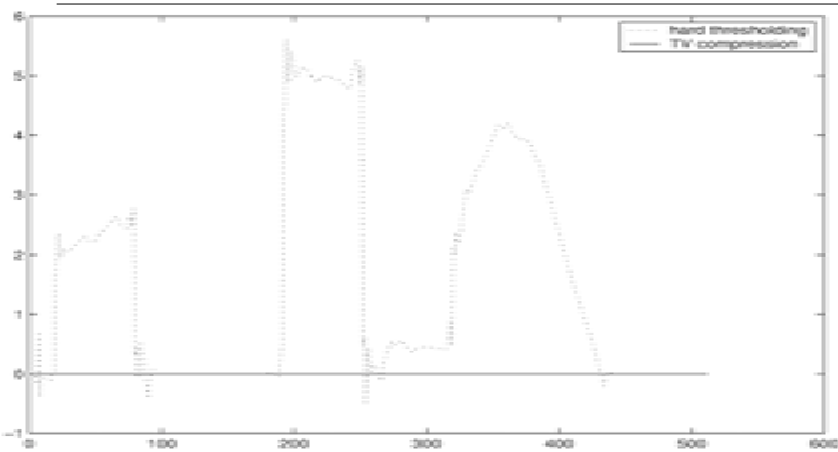
Advantages over TV +Thresh.

¥Reduce the oscillations generated by thresholding

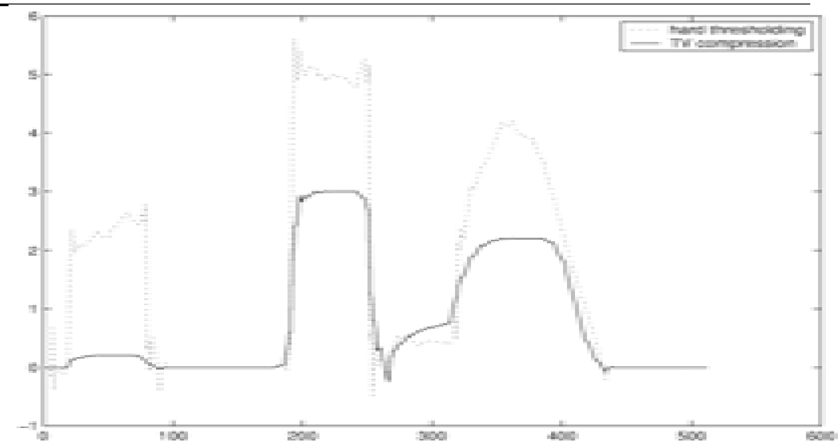
¥May directly operate on wavelets, easiler to combine with comp. schemes.

¥Work on smaller space, can be faster potentially.

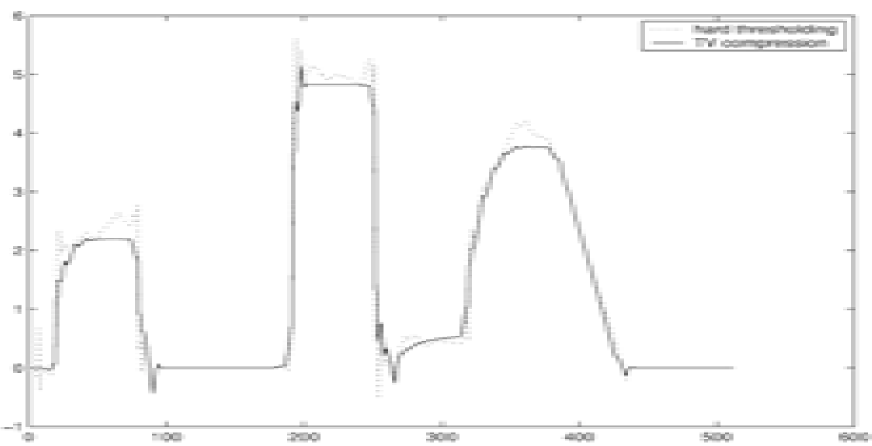
TV Hard Thresholding



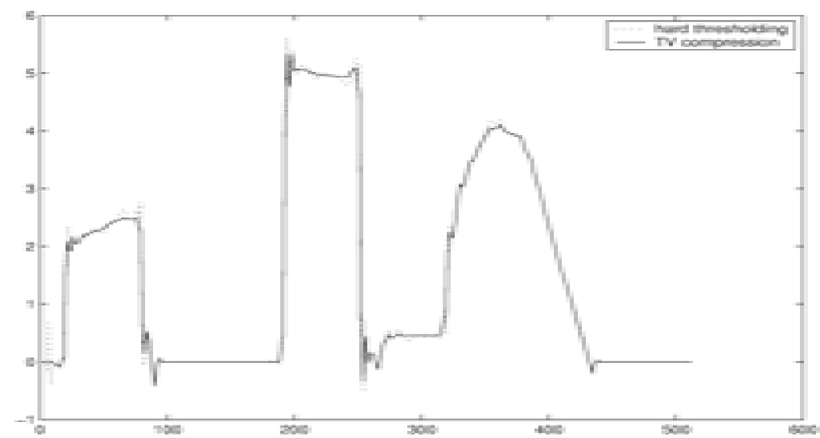
$\lambda = 1$



$\lambda = 0.1$



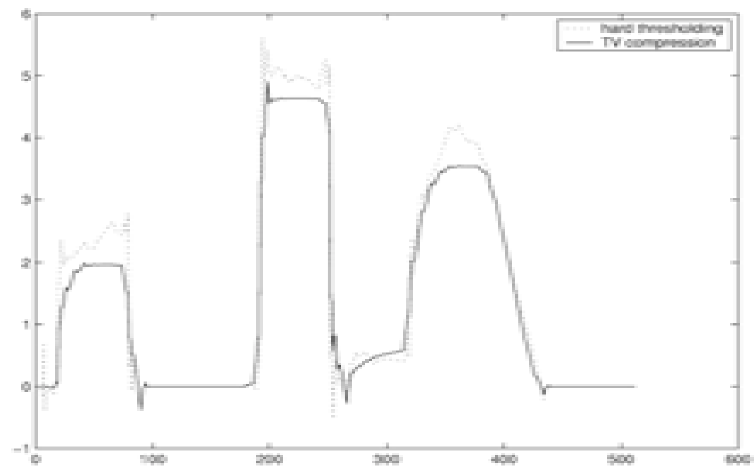
$\lambda = 0.01$



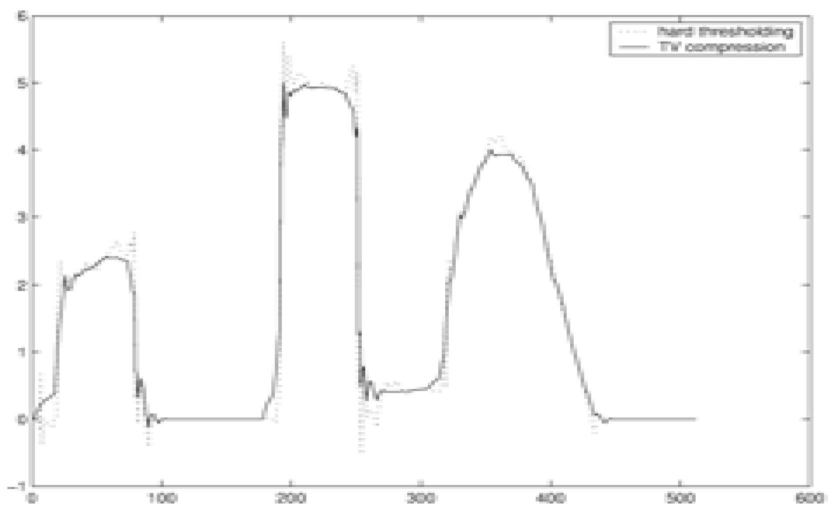
$\lambda = 0.001$

4-level DB4, keep 50 largest coefficients

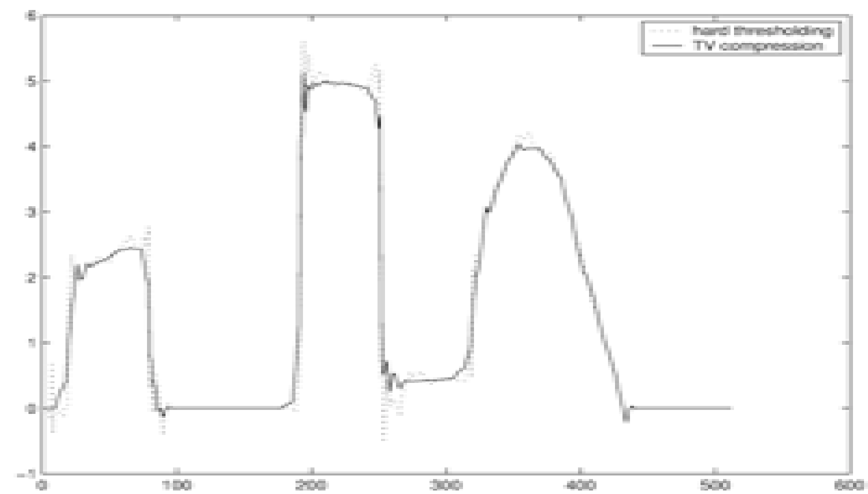
TV Thresholding



Hard Thresh.



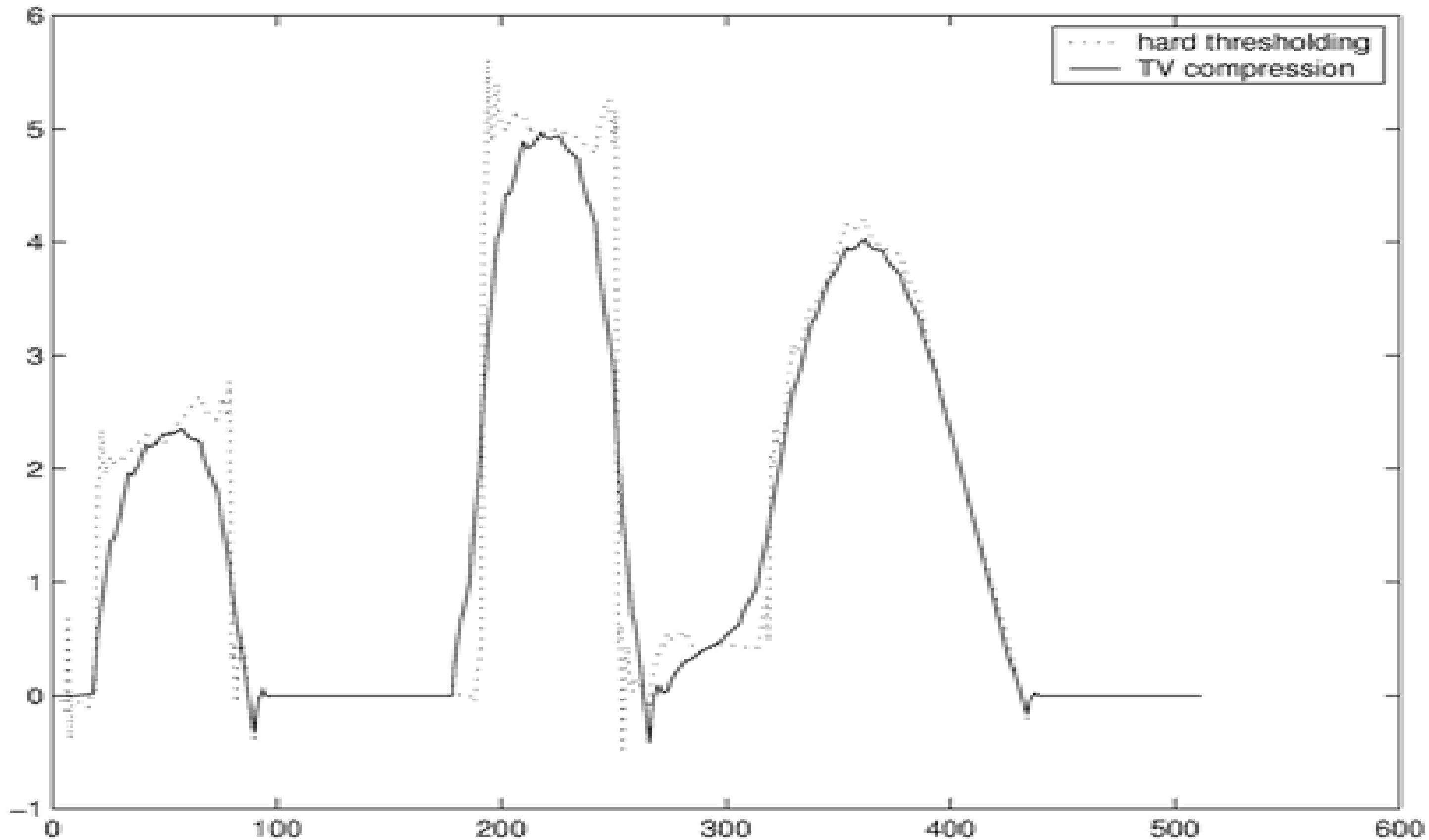
log-function



p-norm

4-level DB4, keep 50 largest coefficients $\lambda = 0.02$

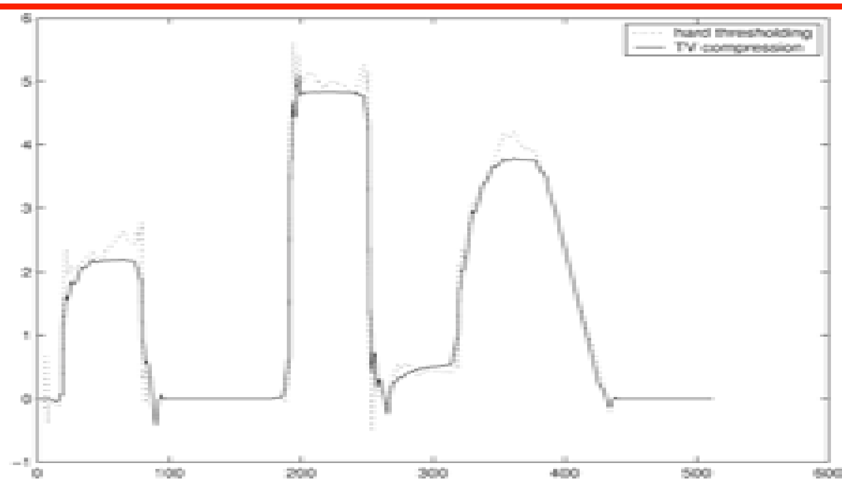
H-1 Regularization



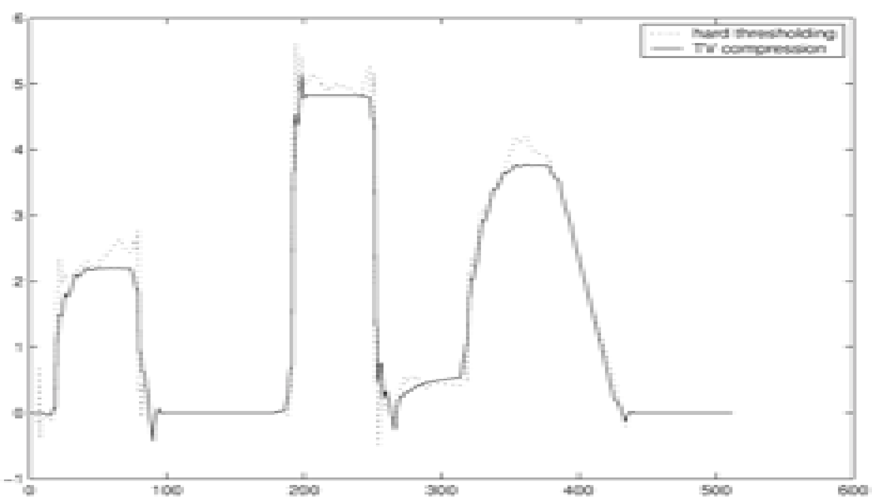
4-level DB4, keep 50 largest coefficients

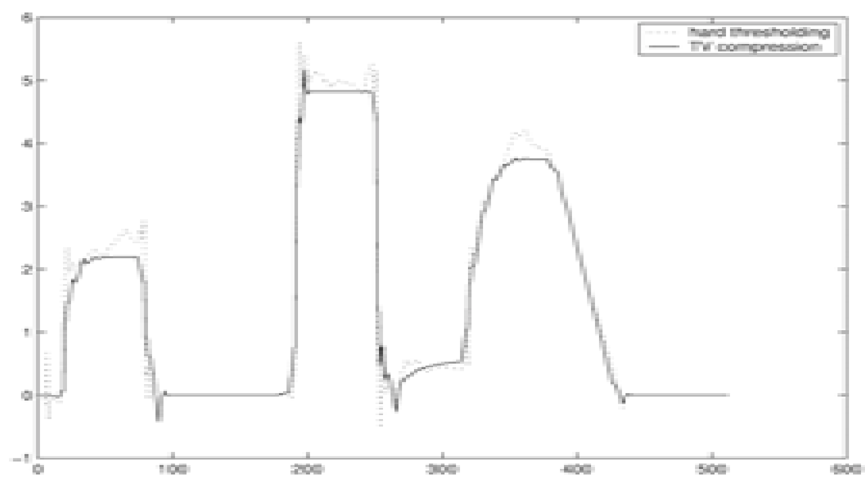
$\lambda = 0.0002$

TV Hard Thresholding



5 iterations

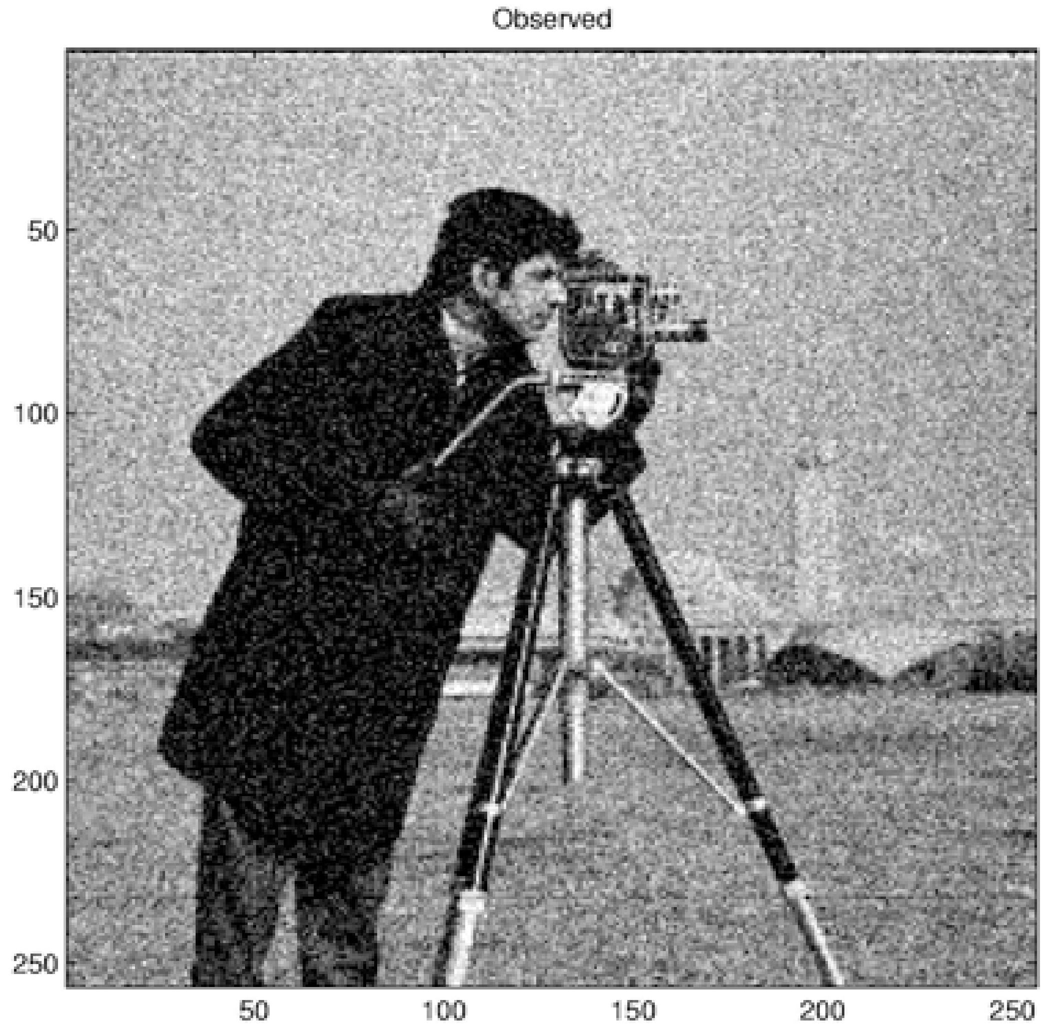




10 iterations 20 iterations

4-level DB4, keep 50 largest coefficients $\lambda = 0.01$

Original Noisy Image



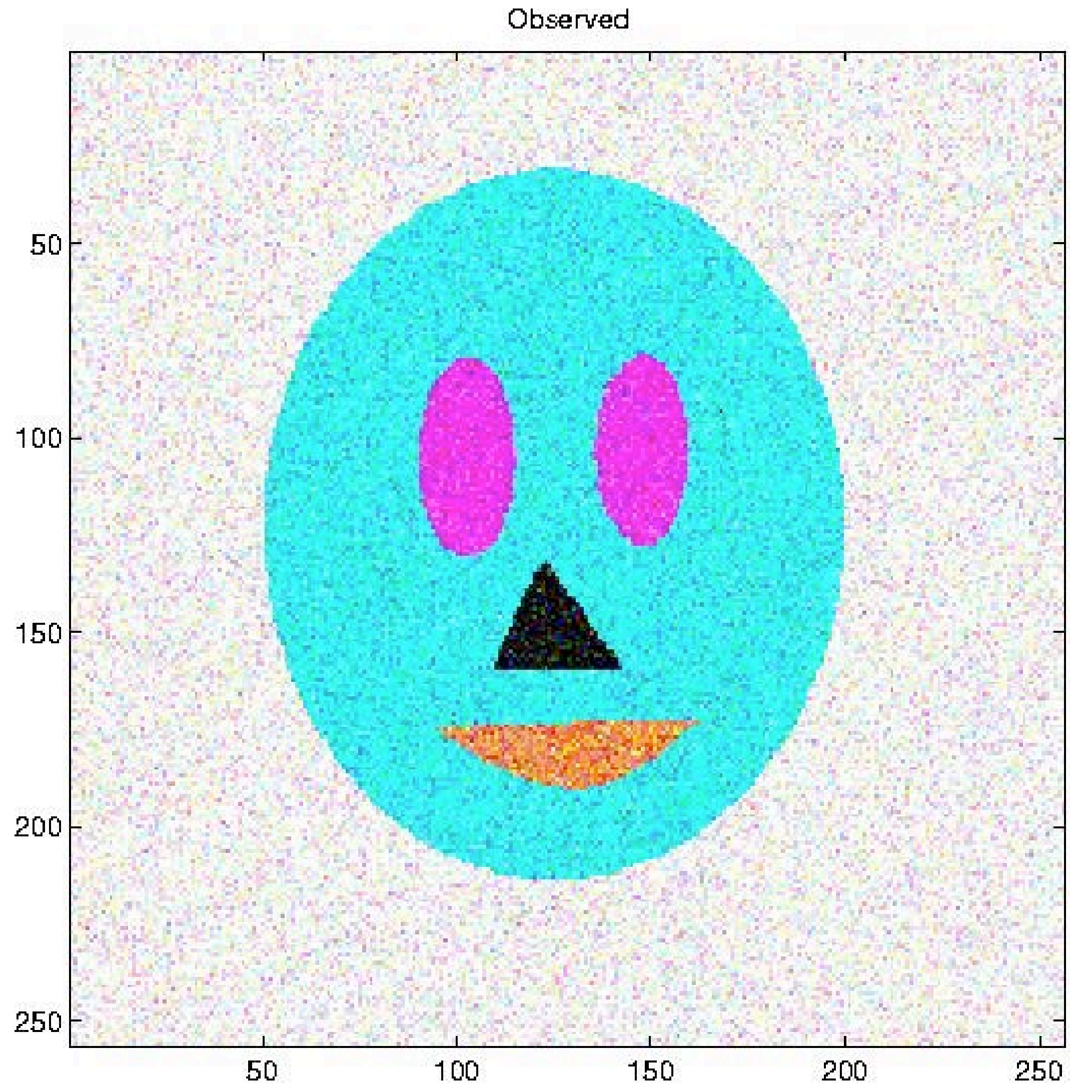
DB6 Hard Thresholding

4-level DB6, Hard thresholding, keep 64x64 coef.

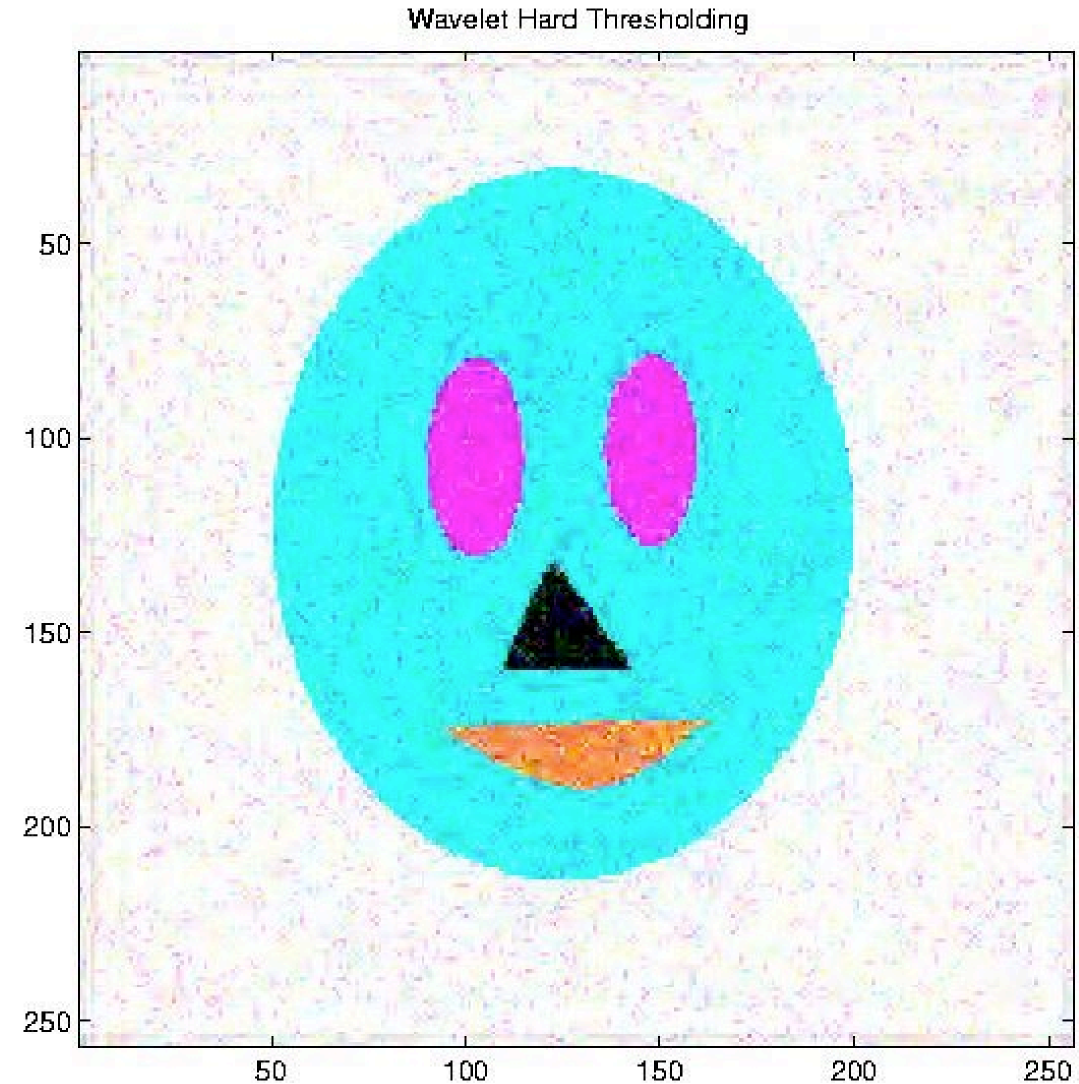
DB6, TV Hard Thresholding

4-level DB6, TV Hard thresholding, keep 64x64 coef.

Original Noisy Image

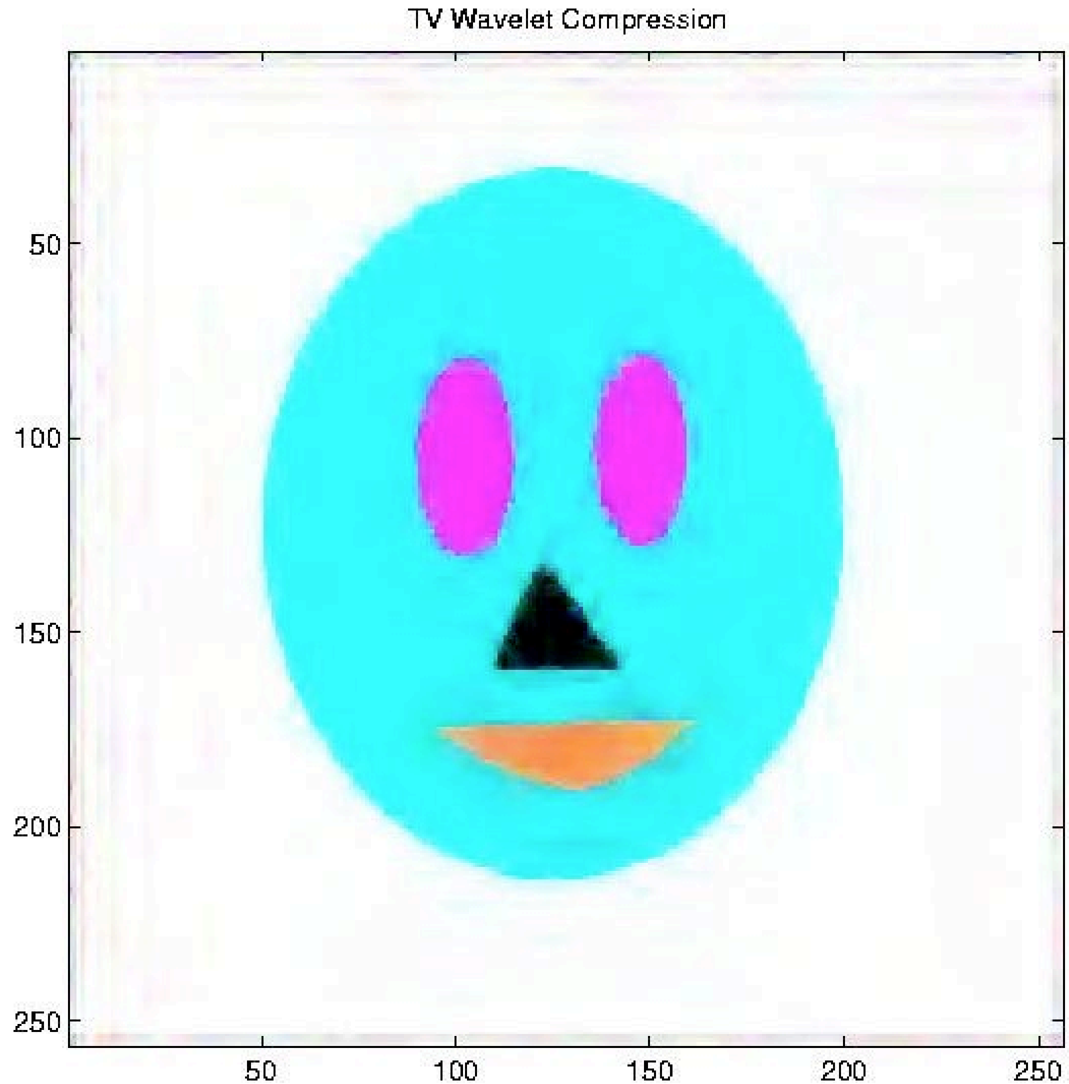


DB6, Hard Thresholding



4-level DB6, Hard thresholding, keep 64x64x3 coef.

DB6, TV Hard Thresholding



4-level DB6, TV Hard thresholding, keep 64x64x3 coef.

Wavelet Image Inpainting

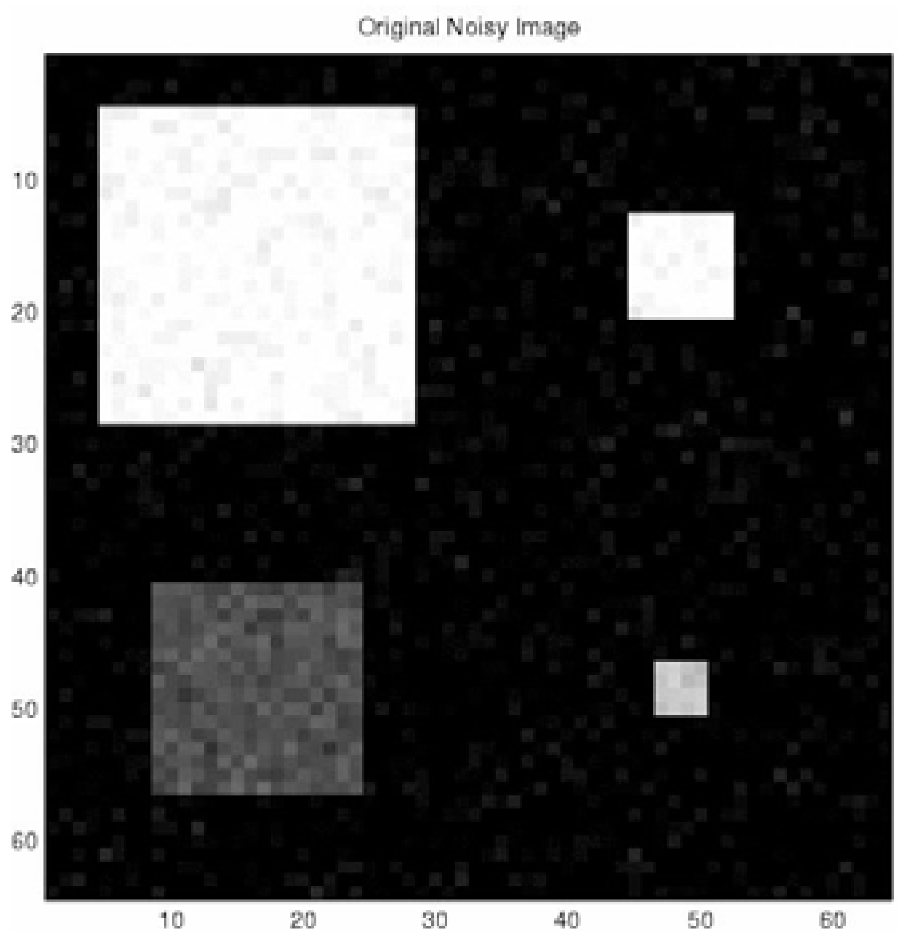
¥Coefficients are damaged or lost in transmission

¥Take to include all coefficients

¥Minimize TV norm s.t. constraints only on retained coefficients, no constraint is imposed on the lost coefficients.

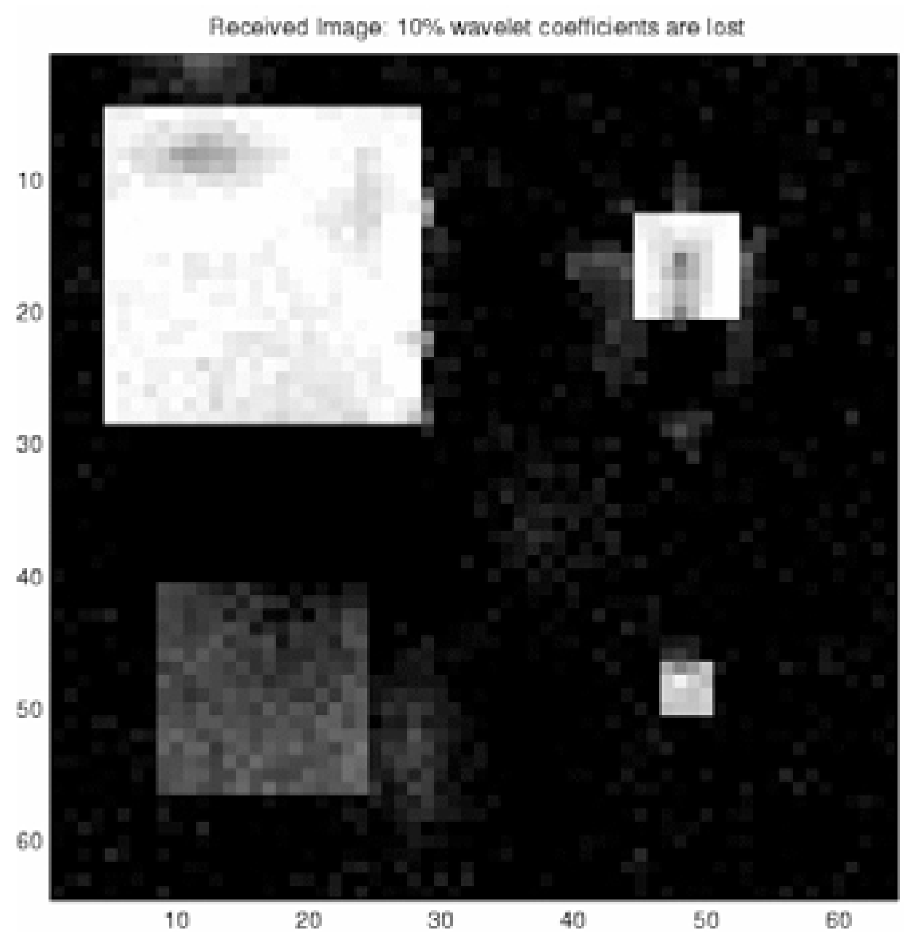
¥Take if is lost, otherwise

Wavelet Image Inpainting



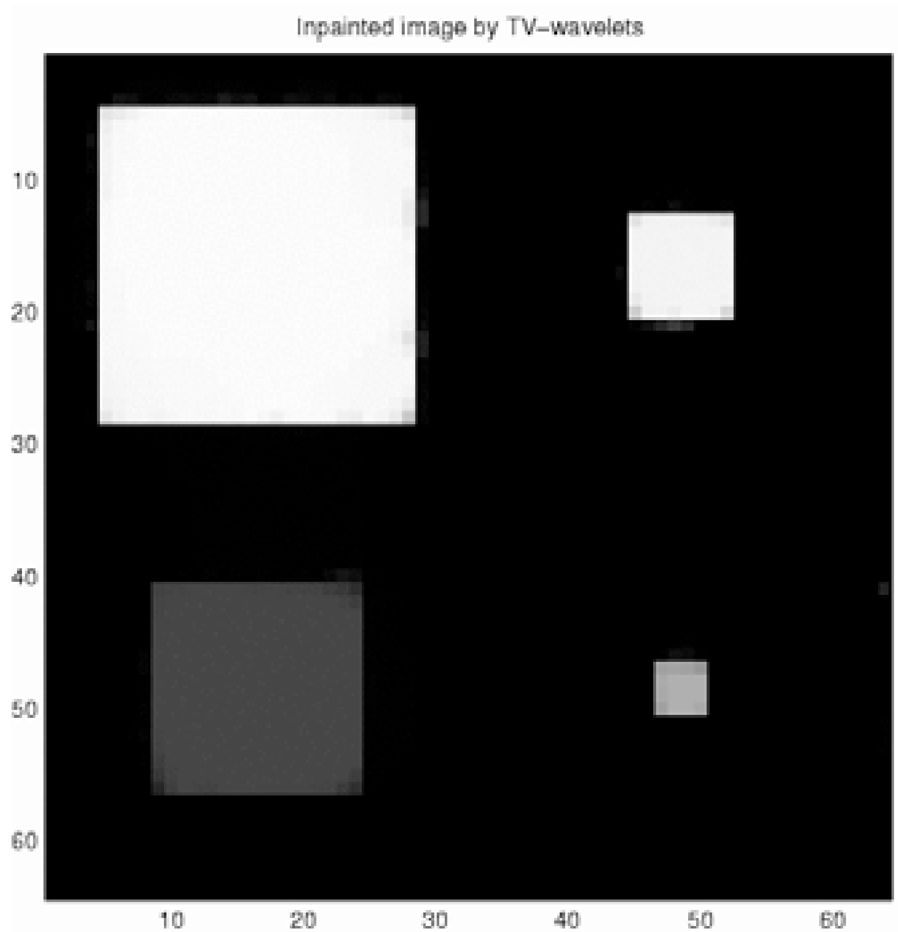
Original noisy test image

Wavelet Image Inpainting



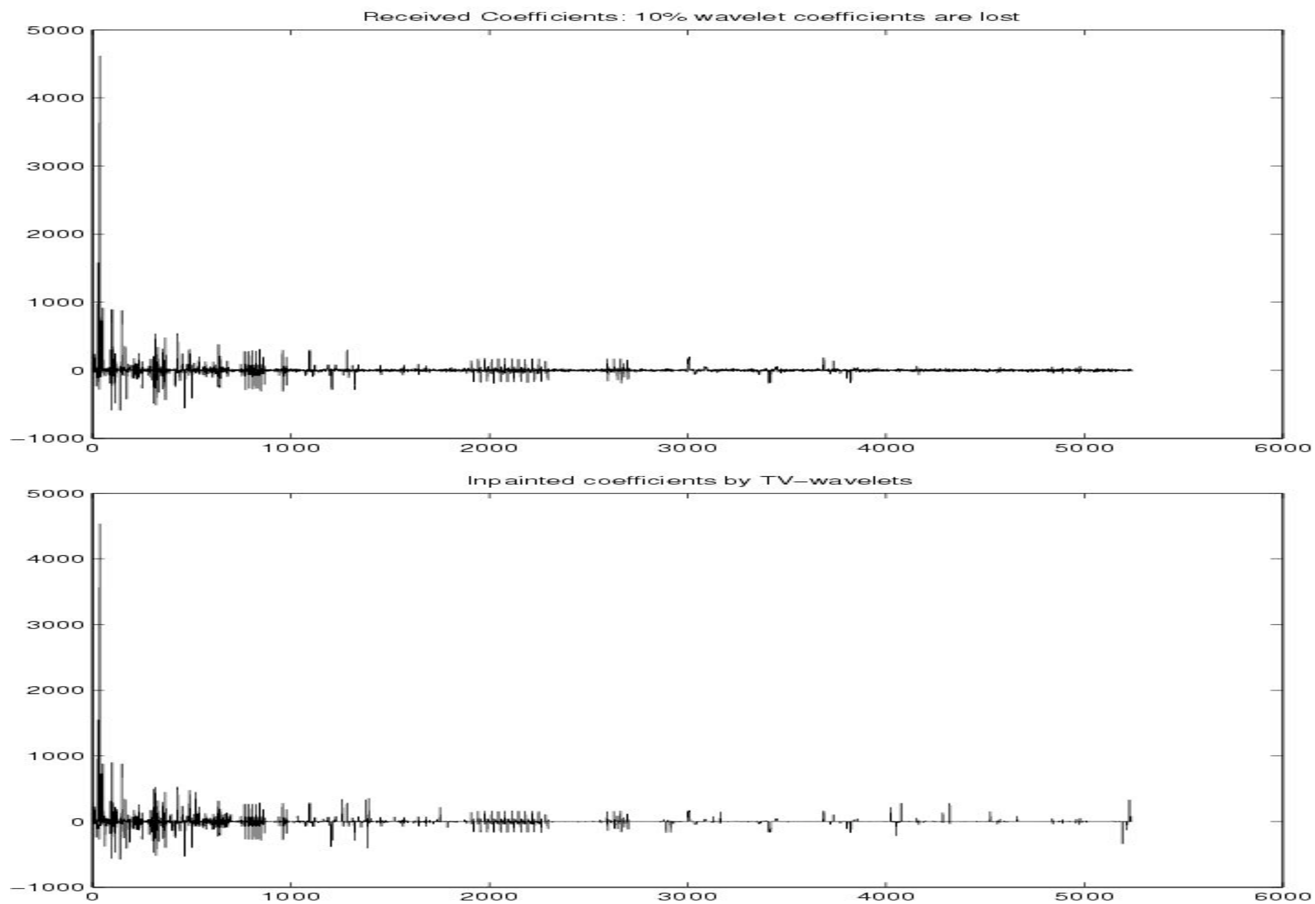
Received Image with 20% coefficients lost, particularly, some edges are damaged

Wavelet Image Inpainting



Inpainted image by minimizing TV norm, it reduced noise while filling in the edge shape

Wavelet Image Inpainting



Wavelet coefficients: lost (top), TV-norm inpainted (bottom)
Certain coefficients are significantly changed to min. TV norm

Conclusions

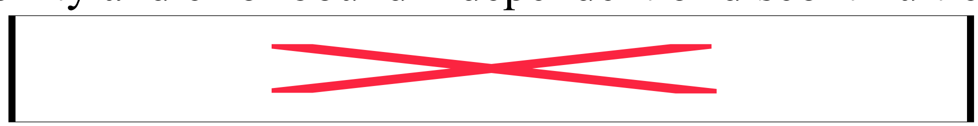
¥Topic 1: ENO-wavelets

¥Goals achieved: satisfies all goals:

¥Essentially Non-Oscillatory

¥Keep pyramidal filtering framework

¥Stability and error bound independent of discontinuities



¥Minimal extra cost (ratio *O(d/n)*) and storage (*O(1)* bit/jump)

¥Generality: Can be applied to other wavelets

¥Application: incorporate ENO-wavelets in the optimization framework of GTW and achieve significant performance gain.

¥Topic 2: TV model for wavelet thresholding

¥Improve the denoising and compression in wavelet thresholding. Reduce the edge oscillations.